Radiation Pressure in Supercritical X-Ray Pulsars

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The accretion of material onto a highly magnetized neutron star results in the formation of an X-ray pulsar. In these objects, the kinetic energy of accreting matter is converted into heat and released in the form of X-rays. If the mass accretion rate is sufficiently high, the luminosity can exceed the Eddington limit and lead to the formation of supercritical X-ray pulsars, where accretion flow is stopped above the neutron star surface. An isotropic point source in the accretion column with a height equal to the radius of the neutron star is considered to emit X-ray photons. The photons incident at various angles with respect to the center of the neutron star and their effect on the atmosphere have been discussed. It is found that the Lorentz force prevents the movement of material across the surface, but the perpendicular component of the radiative force of the incident photon is sufficient to compress the atmosphere of the neutron star.

Introduction

Neutron stars (NSs) are compact objects formed following the supernova of a star with an initial mass of over 8 - 10 solar masses. NSs typically have masses of about $1.4M_{\odot}$ and radii $\sim 10km$. NSs are associated with extremely strong magnetic fields of $\gtrsim 10^{12}G$ at the surface, which makes them the strongest magnets in the Universe[1]. NSs in binary systems can absorb material from their companion in a process known as accretion. In the case of accretion onto a strongly magnetized NS, the system is generally a high mass X-ray binary. The material from the companion O or B type star is prevented from moving across the field lines at the magnetospheric radius[2]. The magnetic field affects the geometry of accretion flow and directs material towards the poles of a star within an area of $\sim 10^{10}cm^2$, leading to the formation of X-ray pulsars (XRPs)[3]. The accretion of plasma along the magnetic field lines leads to the loss of kinetic energy, which is released in the form of X-rays[4]. The characteristic pulsation is caused by a deviation between the magnetic and rotational axes[3,5].

At low mass accretion rates, areas of high luminosity - hotspots or accretion mounds - are created at the surface. When the mass accretion rate is relatively high, the accretion luminosity significantly increases the radiation pressure. Accretion columns are formed when the radiation pressure is large enough to stop accretion flow above the surface of the NS. The size of the column has been shown to be approximately equal to that of the NS's radius[3]. A radiation-dominated shock above the surface leads to the formation of a bright halo by the scattering of X-ray photons. Above the shock, the plasma is in free fall[4].

Accretion columns are supported by the radiation pressure and confined by the strong magnetic field of the NS. When the radiation pressure is large enough to oppose gravity in an accretion column with the shape of a thin wall, the luminosity can largely exceed the Eddington limit[6], leading to the formation of supercritical XRPs. The observed luminosity of these objects has a range of up to $10^{41}ergs^{-1}$. Accretion columns are extended sources of X-ray radiation. However, in the case of relatively low accretion columns, they can be roughly represented by a point source of X-rays located above the surface of a NS because a large fraction of accretion luminosity is emitted by radiation-dominated shock at the top of the accretion column.

Accretion columns illuminate the NS and because of extreme luminosity cause strong radiative forces to be applied to the upper layers of the NS's atmosphere. In this paper, we build on existing models proposed[3] and investigate the effects of flux distribution over the NS surface by decomposing the flux into two components: orthogonal to the NS surface and along the surface of a star. We limit ourselves by the case of flat space-time, which allows us to get analytical expressions for the flux distribution.

Methodology

This paper considers a spherical symmetric NS with a point source located at height h above the NS surface as illustrated in Fig. 1. As the radiation-dominated shock is present at the top of the accretion column, the source is placed $10km$ above the surface, a distance that equals the NS radius. The source has a fixed luminosity of $10^{39}ergs^{-1}$, a value above the Eddington limit which is $\simeq 2 \cdot 10^{38}ergs^{-1}$ for a typical NS[3]. This source can illuminate the NS through the release of photons that are incident in the atmosphere. By considering the arriving photon's radiative force with differing angles respective to the center of the NS, the subsequent effect on the atmosphere is evaluated. The force is resolved into two components, one perpendicular

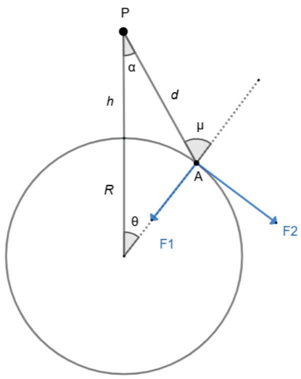

Fig. 1 Illustration of the NS and variables used in calculations. Photons are assumed to be emitted by the point source 'P' and arrive at the surface at 'A'.

to and the other along the surface, and the Lorentz force is calculated to predict any changes to the geometry due to the component along the surface. The following results were arrived at analytically.

1. Coordinates of arriving photon

   Assuming an isotropic point source at height h above the surface of a NS with radius R, the co-latitude θ of the arriving photon with respect to the center of the NS is given by

   $$\cos\theta = \frac{R^2 + (R+h)^2 - d^2}{2R(R+h)}$$

   where

   $$d = \frac{h + R - R\cos\theta}{\cos\alpha}$$

   is the distance from a point source to a given point at the stellar surface, α is the angle between the normal of the point source to the surface and the location of the arriving photon (see Fig. 1). α can be written in terms of θ as

   $$\alpha = \cos^{-1}\left(\sqrt{\frac{-(h+R-R\cos\theta)^2}{2(R\cos\theta)(R+h) - R^2 - (R+h)^2}}\right)$$

2. Arriving flux
   The flux arriving at the NS surface at two different angles is illustrated in Fig. 2. ΔL is the change in luminosity dependent on $\Delta\alpha$. By considering the solid angle for a cone and small angle approximation, it can be defined as

   $$\Delta L = \frac{L(\sin\alpha \cdot \Delta\alpha)}{2}$$

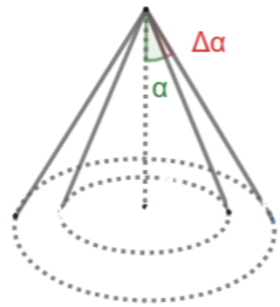

Fig. 2 The angle 'α' and the increment in increase '$\Delta\alpha$'

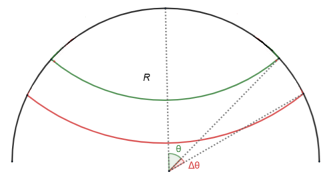

Fig. 3 The angle 'θ' and the increment in increase '$\Delta\theta$'.

Similarly, Fig. 3 displays two angles from the center of the NS.

The radius r of the spherical circle shown by the solid green line is given by

$$r = Rsin\theta$$

ΔS, the change in surface area, is the product of the circumference of the green spherical circle and the distance between both spherical circles in Fig. 3.

$$\Delta S = 2\pi R\sin\theta \cdot R\Delta\theta$$

Therefore, the flux due to the arriving photon can be written as the ratio between ΔL and ΔS as

$$F = \frac{L\sin\alpha}{4\pi R^2\sin\theta} \cdot \frac{\Delta\alpha}{\Delta\theta}$$

where

$$\frac{\Delta\alpha}{\Delta\theta} \approx \left[\frac{\cos\alpha(R+h)}{\sqrt{R^2 - \sin^2\alpha(R+h)^2}} - 1\right]^{-1}$$

This equation for flux is plotted in Fig. 4 in terms of the angle θ for the aforementioned parameters. When the height of the column was varied in similar studies[3], the resulting graphs were horizontally dilated. Fig. 4 was also

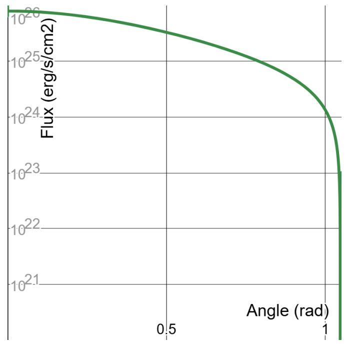

Fig. 4 The distribution of flux over the surface of the NS, illuminated by a point source in the accretion column 10 km above the surface. Parameters: $M = 1.4M_{\odot}$, $R = 10\,\text{km}$.

compared with flux distribution in curved space-time[3] and both plots are largely similar.

The flux can be divided into components, one perpendicular to (F_1) and one along the surface (F_2) of the NS.

$$F_1 = F\cos\mu$$
$$F_2 = F\sin\mu$$

where $\mu = \theta + \alpha$

3. Radiation Pressure

For an absorbing surface, the radiation pressure can be given by

$$P_{\text{rad}} = \frac{F}{c}$$

where P_{rad} is the radiation pressure, F the flux, and c the speed of light.

Similarly, for a reflecting surface, radiation pressure is given by

$$P_{\text{rad}} = \frac{2F}{c}$$

To account for the transferred energy during the reflection of the photon from the NS surface, an absorption constant, a, is included and varied from $[0,1]$.

$$P_{\text{rad}} = \frac{(2-a)F}{c}$$

The radiation pressure can also be resolved into components by substituting F_1 and F_2 in the above equation:

$$P_{\text{rad}_1} = \frac{(2-a)F_1}{c}$$
$$P_{\text{rad}_2} = \frac{(2-a)F_2}{c}$$

4. Force per electron due to the flux along the surface of the NS

Assuming a surface density[3] of $0.35gcm^{-2}$ the number of electrons per cm^2 was found assuming it was equal to the number of protons.

$$N_{e^-} = \frac{0.35}{m_p}$$
$$N_{e^-} = 2.1\cdot 10^{23}$$

The force per electron can be determined by finding the ratio between the maximum radiative pressure and the number of electrons.

5. Calculation of Lorentz Force

The relativistic expression for kinetic energy was considered:

$$E_K = (\gamma - 1)mc^2$$

where $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$

This expression can be rearranged for velocity:

$$v = c\sqrt{1 - \left(\frac{mc^2}{E_K + mc^2}\right)^2}$$

For an electron with $E_K = 1\,\text{keV} = 1.602\cdot 10^{-9}\,erg$ and $m = 9.11\cdot 10^{-28}\,g$,

$v \approx 1.873\cdot 10^9\,cms^{-1} \approx 0.0624c$

The Lorentz force when $E = 0$ is given by:

$$F_L = q\cdot\left(\frac{v}{c}\times B\right)$$

The charge for an electron is $4.8032\cdot 10^{-10}\,\text{statC}$ and the magnetic field strength was taken as $10^{12}\,G$ as for a typical XRP.

$$F_L \approx 30\,dyn$$

Results

Fig. 5 is plotted from the final equations for the components of flux resolved in Sect. 2 of the methodology.

The flux perpendicular to the surface is maximum exactly beneath the accretion column and decreases as we move towards the equator. In contrast, the force along the surface is a minimum below the column and increases to a maximum at $0.338\,\text{rad}$. Both components are 0 at $\sim 1.05\,\text{rad}$, after which the photons are not incident on the atmosphere in non-relativistic conditions. The total flux was used to find the radiation pressure,

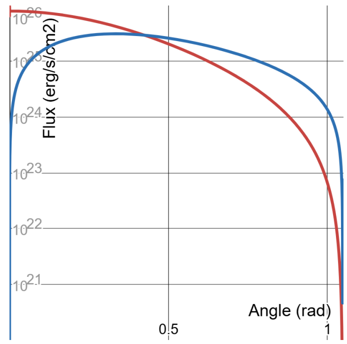

Fig. 5 The components of flux over the surface of the NS. Flux perpendicular to the surface (F_1) is represented by the red solid line and the flux along the surface (F_2) by the blue solid line. Parameters: $M = 1.4 M_{\odot}$, $R = 10\,km$.

which reached a maximum of $2.08 \times 10^{15}\,dyn \cdot cm^{-2}$. The force per electron is therefore $9.9 \times 10^{-9}\,dyn$. In comparison to the Lorentz force of $30\,dyn$, the force due to the flux is significantly less.

As there are many magnitudes of difference between the Lorentz force and the force of the incident flux on an electron, we do not expect that radiative force applied to the upper layers of the neutron star atmosphere will result in motion of material across the surface of a NS even in the case of highly super-Eddington X-ray pulsars. If material were to be moved towards the equator, a magnetic field strength of $\sim 3 \times 10^2$ G would be required for the case of an accretion column of $10\,km$ height and total luminosity $\sim 10^{39}\,erg \cdot s^{-1}$. The flux perpendicular to the surface of the NS can be sufficiently larger than the local Eddington flux (which is $\sim 10^{25}\,erg \cdot s^{-1} \cdot cm^{-2}$). Under this condition, the atmosphere can be effectively compressed by the external source of X-ray radiation.

Although the results were arrived at for a fixed luminosity of $10^{39}\,erg \cdot s^{-1}$, the methodology used is applicable to other high luminosities. The resulting plot would be similar but scaled to match the new luminosity.

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