

Is it Too Late Now to Say Sorry? Mathematical Proofs of Conflict Resolution Strategies on Social Media

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This paper looks at how conflicts may be resolved under various circumstances. We begin by examining mathematical structures of conflict and utility functions, applying a preference structure to the coordination process in a conflict, and finally analyzing conflict in terms of recurrence relationships and culminating in an overview of a stochastic process. We present two possible frameworks for defining conflict and offer a conflict-resolution strategy for each framework.

Introduction

History was shaped by conflict - born every time multiple self-serving agents pursued opposing interests and rarely conceded or accepted a compromise. As a result, all parties were left unsatisfied (a lose-lose situation) or faced massive consequences, including loss of life, resources, time, or increased risk.

Take the Cuban Missile Crisis, for instance. When two self-serving agents (the United States and the Soviet Union) pursuing opposing interests (Dominance as a world power) refused to cooperate (reduce military aggression), the missile crisis was born. As a result of this conflict, both parties faced the consequence of increased risk of nuclear armageddon.

Today's world is also one of conflicts. Countries that cannot agree on peace or a collaborative environmental policy may face the consequences of war or global warming. Political parties that refuse to cooperate in pursuit of party dominance will fail to pass key legislation, suffering national economic losses or struggling to respond to emergency situations. Individuals who quarrel to assert their dominance will risk a physical confrontation that can result in injury for everyone.

As shown, self-serving actors who fail to concede or cooperate will risk a catastrophic result in which all parties lose. Thus, it is a global priority to produce a strategy that can help resolve conflicts to encourage agents to pursue a win-win solution that can benefit everyone. Theoretically, such a strategy would enable greater collaboration at all levels, from the interpersonal to the international. Applied at a global scale, an effective conflict resolution strategy would expedite the resolution of major challenges, including climate change, disease, wars, and political polarization.

This paper focuses on a rational strategy for quickly resolving a dispute in social media comment sections. The goal is to allow all parties involved in the conflict to leave without animosity toward any other party.

The Role of Game Theory in Conflict Resolution

Game theory, a mathematical framework for analyzing strategic interactions among rational agents, has been pivotal in understanding and resolving conflicts. A fundamental aspect of game theory is the concept of the *Nash equilibrium*¹. This is a state in which no player can improve their outcome by unilaterally changing their strategy. When applied to conflict resolution, the Nash equilibrium provides a theoretical basis for predicting outcomes where all parties have an incentive to cooperate.

Consider the classic *Prisoner's Dilemma*, a game in which two players can either cooperate or defect. If both cooperate, they achieve a win-win outcome. However, if one defects while the other cooperates, the defector gains a larger reward. This structure leads to a dilemma, as rational players are tempted to defect, resulting in a lose-lose outcome. By applying strategies such as repeated interactions or enforcing external incentives, the players can achieve cooperation².

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \quad \forall s_i, s_i^* \in S_i$$

where u_i represents the utility of player i , s_i^* the strategy of player i , and s_{-i}^* the strategies of all other players.

Conflict Resolution in Social Media

Social media platforms often become breeding grounds for conflict due to anonymity and the lack of face-to-face accountability. A game-theoretic approach can be used to model disputes in comment sections as repeated games. Each user can choose to escalate (E) or de-escalate (D) the conflict. A possible payoff matrix is shown below:

	E	D
E	$(-1, -1)$	$(1, -2)$
D	$(-2, 1)$	$(0, 0)$

Here, the cooperative outcome (D, D) ensures no escalation and neutral interactions. To encourage such outcomes, platforms can implement systems that reward de-escalation, such as highlighting constructive comments or penalizing abusive behavior³.

Advanced Mechanisms for Conflict Resolution

Recent studies suggest that implementing reputation-based systems can effectively reduce hostility in online interactions. These systems, inspired by the Folk Theorem in game theory, use historical behavior to influence current incentives⁴. For example, users with a history of de-escalating conflicts may gain social credibility, encouraging others to cooperate in future disputes.

Artificial intelligence and machine learning also play a growing role in moderating conflicts. Algorithms trained to detect aggressive language can intervene early by either issuing warnings or restricting further escalation⁵. Such proactive moderation creates an environment where users feel safer, reducing the likelihood of repeated hostile encounters.

Additionally, insights from evolutionary game theory suggest that promoting altruistic behavior among users can foster a cooperative culture. By rewarding individuals who actively mediate or resolve disputes, platforms can reduce overall conflict prevalence⁶. Altruistic strategies have been observed to increase social trust and enhance the collective utility of online communities.

Mathematical Preliminaries

In any game theoretic construct, the primary goal is survival in some form. Even in Darwinian terms, there is variation and selection working together to produce adaptation. In any circumstance, we are usually dealing with expected maximal returns from a player's point of view. This would exist even in the case of mixed strategies, which are normally used to show that outcomes exist for zero-sum games with two players. In this regard, we have the Minimax theorem of Von Neumann from 1928. Suppose $A = (a_{ij}), i = 1, 2, \dots, n; j = 1, 2, \dots, m$ is the payoff matrix for a zero-sum two-person game with $p = (p_1, p_2, \dots, p_n)$ being the probability vector for player I choosing action i , and likewise $q = (q_1, q_2, \dots, q_m)$ be the probability vector for player II, then for this two-person, zero-sum game with payoff matrix A , there exist those probability vectors p^* and q^* such that

$$V(p^*, q^*) \equiv \max_p \min_q V(p, q) = \min_q \max_p V(p, q) \quad (1)$$

where $V(p, q) \equiv (p, Aq)$, which is a way of saying that the payoff to player I using strategy p is the negative of the payoff to player II using strategy q .

Here $V(p^*, q^*)$ is called the value of the game. Note that the Minimax theorem only states the existence of such strategies but does not state the means of finding them. We will use the structure of⁷. Here we are using the premise of an information system, which is a pair $S = (U, A)$, where U is a nonempty, finite set called the *universe* and elements of U are called *objects (agents)*, and A is a non-empty finite set of *attributes(issues)*. Every attribute $a \in A$ is a function: $a : U \rightarrow V_a$, where V_a is the set of *values* of a , called the *domain* of a . The elements of V_a are called *opinions*, which means $a(x)$ represents the opinion of agent x about issue a . In conflict analysis, it is assumed for simplicity that the set

$V_a = \{-1, 0, 1\}$, for every value of a , which represents *against, neutral, favorable* respectively. It can also be written as $V_a = \{-, 0, +\}$. In order to express relations between agents, three basic binary relations are used: *conflict, neutrality, alliance*. In support, one defines the function $\phi_a(x, y)$ as follows:

$$\phi_a(x, y) = \begin{cases} 1, & \text{if } a(x)a(y) = 1 \text{ and } x = y \\ 0, & \text{if } a(x)a(y) = 0 \text{ and } x = 0 \text{ or } y = 0 \\ -1 & \text{if } a(x)a(y) = -1 \text{ and } x \neq y \end{cases} \quad (2)$$

For instance, if $\phi_a(x, y) = 1$, this means agents x and y have the same opinion about issue a as in they are *allied* on issue a . If $\phi_a(x, y) = 0$, this means that at least one agent x or y has a neutral approach to issue a as in they are neutral on the issue a , and finally, if $\phi_a(x, y) = -1$, this means they have different opinions on issue a i.e. they are in conflict on a . Next, the three basic relations R_a^+, R_a^0, R_a^- over U^2 are called alliance, neutrality and conflict. These are defined as follows:

$$R_a^+(x, y) \text{ iff } \phi_a(x, y) = 1 \quad (3)$$

$$R_a^0(x, y) \text{ iff } \phi_a(x, y) = 0 \quad (4)$$

$$R_a^-(x, y) \text{ iff } \phi_a(x, y) = -1 \quad (5)$$

The alliance relation $R_a^+(x, y)$ is an equivalence relation as in

1. $R_a^+(x, x)$
2. $R_a^+(x, y)$ implies $R_a^+(y, x)$
3. $R_a^+(x, y)$ and $R_a^+(y, z)$ implies $R_a^+(x, z)$

These equivalence classes of alliance are called coalitions. Likewise, we have the following for the conflict relationship:

1. Non $R_a^-(x, x)$
2. $R_a^-(x, y)$ and $R_a^-(y, z)$ implies $R_a^+(x, z)$
3. $R_a^-(x, y)$ and $R_a^+(y, z)$ implies $R_a^-(x, z)$

The idea of Coalitions

With $a \in A$, if there exists a pair (x, y) such that $R_a^-(x, y)$ then one states that the attribute a is conflicting, otherwise the attribute is conflictless. If a is a conflictless attribute, then R_a^+ has the equivalence classes defined by $X_a^+ = \{x \in U : a(x) = +\}$, $X_a^- = \{x \in U : a(x) = -\}$.

1 The existence of the Von Neumann-Morgenstern Utility function

Based on the work of John von Neumann and Oskar Morgenstern, four axioms have to be satisfied in order for an individual to have a utility function. We look at why these axioms must work in our scenario. First, we state the axioms:

1. Axiom 1: Completeness.
2. Axiom 2: Transitivity.
3. Axiom 3: Continuity.
4. Axiom 4: Independence.

Axiom 1: Completeness

The first criterion that the Von Neumann–Morgenstern utility theorem presents is completeness. This axiom argues that for a utility function to exist, a Decision Maker (DM) must have a preference or indifference between two choices, L and M. For instance, a DM must prefer Apples over Bananas, establishing preference. Alternatively, the DM must prefer Apples over Bananas while preferring Bananas over Apples, establishing equal preference or indifference.

This can be represented formally as saying that for any lotteries L and M , at least one of the following holds:

$$L \succsim M, M \succsim L \quad (6)$$

In other words, the individual must express some preference or indifference, and this implies reflexivity.

Axiom 2: Transitivity

A DM's preferences must demonstrate transitivity. Let us return to the example of fruits. Suppose a DM prefers oranges over bananas and prefers apples over oranges. In that case, the DM must prefer apples over bananas. This idea of transitivity is the same idea found in integer variables; if $Y > Z$ and $X > Y$, $X > Z$. Mathematically, we write this as:

$$\text{If } L \succsim M \text{ and } M \succsim N, \text{ then } L \succsim N \quad (7)$$

Axiom 3: Continuity

The axiom of continuity asserts that for any two choices, there must be a continuum of choices between them with a 'middle' level of preference. For instance, between the choice of Apples and Bananas, the choice of Oranges, Dragon fruits, and Kiwis exist; all three preferred more than bananas but less than apples. Mathematically, this means

$$\text{If } L \succsim M \succsim N,$$

then there exists a probability $p \in [0, 1]$ such that

$$pL + (1 - p)N \sim M \quad (8)$$

The continuity axiom really states that if there are three lotteries, which we can designate as A, B, C with $A \succ B$ and $B \succ C$, then there is a $p \in [0, 1]$ such that $pA + (1 - p)C \sim B$. This means that there is a compound lottery involving the best A and the worst C , which the individual will regard as indifferent to B , which is the intermediate. Geometrically, this simply means that the indifference curve through B (the intermediate) must cross the line segment connecting A (the best) and C (the worst).

Axiom 4: Independence

The *Independence Axiom* states that if A, B, C are any three lotteries, and if $p \in [0, 1]$, then

$$pA + (1 - p)C \succsim pB + (1 - p)C \quad (9)$$

if and only if

$$A \succsim B \quad (10)$$

In order to understand this axiom, we first look at what compound lotteries are. We look at when decision-making takes place under risk. In this case, a basic assumption is in the direction of the axiom of reduction of compound lotteries. A lottery can be interpreted as running a random device, whereas a compound lottery is equivalent to running a random device, such that the results are themselves lotteries. This is similar to a two-step uncertainty process. One way to define such a process is to assume that the probability process is now $(0, 1] \times (0, 1] = (0, 1]^2$, with a uniform distribution on the unit square. Using⁸, the compound lottery distribution can be generated by taking $E = (0, \alpha]$ and considering a random variable $Z : (0, 1]^2 \rightarrow \mathbb{R}$ defined on the unit square in the following manner:

$$Z(\omega_1, \omega_2) = \begin{cases} 5, & \text{if } \omega_1 \in (0, \alpha], \omega_2 \in (0, \frac{3}{4}) \\ 10, & \text{if } \omega_1 \in (0, \alpha], \omega_2 \in (\frac{3}{4}, 1) \\ 10, & \text{if } \omega_1 \in (\alpha, 1], \omega_2 \in (0, \frac{1}{3}) \\ 20, & \text{if } \omega_1 \in (\alpha, 1], \omega_2 \in (\frac{1}{3}, 1) \end{cases} \quad (11)$$

We will show that Axiom 4 does not need to hold for our specific application. For this purpose, we will refer to⁹.

The expected utility hypothesis, has been known to depend on the independence axiom, which means if one were to bypass this axiom, it has to be replaced by some other structure. As remarked in⁹, "a risky prospect A is weakly preferred (i.e., preferred or indifferent) to a risky prospect B if and only if a p:(1-p) chance of A or C respectively is weakly preferred to a p:1-p chance of B or C" with p being a positive probability, and risky prospects being A,B,C. The other axioms help to establish the existence of a continuous preference function based on probability distributions.

In order to see how this matters, we start with a definition. If $F(\cdot)$ and $F^*(\cdot)$ are two cumulative distribution functions over a wealth interval $[0, M]$, then F^* is said to differ from F by a simple compensated spread if the individual is indifferent between F and F^* , and if $[0, M]$ may be partitioned into disjoint intervals I_L and I_R with I_L to the left of I_R such that $F^*(x) \geq F(x)$ for all x in I_L and $F^*(x) \leq F(x)$ for all $x \in I_R$. In order to see how one could still use the concepts of expected utility theory, one starts by considering the choice set as the set $D[0, M]$ of all probability distributions $F(\cdot)$. One also assumes that the individual's preference ranking over this set is complete, transitive, and representable by a real-valued preference functional $V(\cdot)$ on $D[0, M]$. The key here is a continuity of preferences, which means one places the topology of weak convergence that allows defining a sequence $F_n(\cdot) \subset D[0, M]$ as converging to $F(\cdot)$ if and only if $F_n(x) \rightarrow F(x)$ at each point of continuity x of $F(\cdot)$. In this regard,⁹ considers the following sequences as convergent:

- Density functions involved in pointwise convergence
- A sequence of distributions can collapse to a degenerate distribution $G_c(\cdot)$. It is assumed that this assigns a unit mass to the point c
- The sequence $G_{c_n} \rightarrow G_c(\cdot)$ where $c_n \rightarrow c$
- In terms of distributions if $\int g(x)dF_n(x) \rightarrow \int g(x)dF(x)$ for all continuous $g(\cdot)$ on $[0, M]$, this implies convergence in distribution of $F_n(\cdot)$ to $F(\cdot)$. The weak convergence topology is the coarsest topology where the expected utility functional $\int U(x)dF(x)$ is continuous for all continuous $U(\cdot)$ on $[0, M]$

Besides the famous Allais paradox, a second type of violation of the independence axiom is the entire idea of subjective expected utility models (SEU). As seen in⁹, these models assume that an individual transforms called the set of objective probabilities p_i of a risky prospect into their own subjective probabilities $\pi(p_i)$. These are called decision weights. They then try to do some sort of maximization $\sum_i U(x_i)\pi(p_i)$, where p_i is the probability of outcome value x_i . The independence axiom requires $\pi(p_i)$ to be linear empirical estimates, but in reality, studies have found that individuals overemphasize small probabilities and underemphasize large probabilities. In other

words, this is clearly not a linear system in real life. In order to bypass the issues of the independence axiom, one needs the real-valued preference functional $V(\cdot)$ to be differentiable, and a norm needs to exist on the space under consideration $\Delta D[0, M]$, which is defined as $\lambda(F^* - F)|F, F^* \in D[0, M], \lambda \in \mathbb{R}^1$.

The weak convergence topology on $D[0, M]$ allows the L^1 metric $d(F, F^*) = \int |F^*(x) - F(x)|dx$ which induces the norm

$$\|\lambda(F^* - F)\| \equiv |\lambda| \cdot d(F, F^*) \tag{12}$$

the equation being true on $\Delta D[0, M]$ This is a way of saying that preference functional $V(\cdot)$ is smooth in the sense that it is Frechet differentiable on the space $D[0, M]$ with respect to the norm $\|\cdot\|$. A function $V(\cdot)$ is Frechet differentiable at a point F in $D[0, M]$ if there exists a continuous functional $\psi(\cdot; F)$ defined on $\Delta D[0, F]$ such that

$$\lim_{\|F^* - F\|} \frac{|V(F^*) - V(F) - \psi(F^* - F; F)|}{\|F^* - F\|} \tag{13}$$

Another representation of Frechet Differentiability is:

$$V(F^*) - V(F) = \psi(F^* - F; F) + o(\|F^* - F\|) \tag{14}$$

Here $o(\cdot)$ denotes a function that is zero at zero and of a higher order than its argument.

We recall two results in this regard. Suppose X is a normed linear space. One question that arises is whether there are enough continuous linear functionals on X to separate the points of X . The answer is in the affirmative, and we have the Hahn Banach theorem for normed linear spaces: Let X be a real or complex normed linear space, let $M \subseteq X$ be a linear subspace, and let $l \in M^*$ be a bounded linear functional on M . Then there exists a linear functional $\tilde{l} \in X^*$ that extends l and in addition, $\|\tilde{l}\|_{X^*} = \|l\|_{M^*}$.

Next, we recall that the norm is induced by the inner product. For a general inner product on a space V , define the norm $\|v\| = \sqrt{\langle v, v \rangle}$ which obeys the three properties:

1. Positiveness: $\|v\| > 0$ iff $v \neq 0$
2. Homogeneity in scaling: for all vectors $v \in V$, and all scalars λ in its base field, $\|\lambda v\| = |\lambda| \|v\|$, where $|\lambda|$ represents the absolute value or modulus
3. Triangle inequality: for all vectors $u, v \in V$, $\|u + v\| \leq \|u\| + \|v\|$

A function from a vector space to its base field is called a functional. A linear functional ϕ on a normed vector space V is called bounded if there exists some real M such that $\|\phi(v)\| \leq M \|v\|$ for all $v \in V$. The result required is the Riesz representation theorem, which states that for a continuous linear functional ϕ on a Hilbert space V , there exists a unique $u \in V$

such that $\phi(v) = \langle u, v \rangle$ for all $v \in V$, and $\|u\| = \|\phi\|$. Now, using the Hahn-Banach theorem and the Riesz Representation Theorem, we see that a linear extension of $\psi(\cdot; F)$ to $L^1[0, M]$ may be constructed, and further, on $L^1[0, M]$, for any $F^* \in D[0, M]$,

$$\psi(F^* - F; F) = \int (F^*(x) - F(x))h(x; F)dx = - \int (F^*(x) - F(x))dU(x; F) \tag{15}$$

where $h(\cdot; F) \in L^\infty[0, M]$ and $U(x; F) \equiv - \int_0^x h(s; F)ds$. From here it is clear that $U(\cdot; F)$ is absolutely continuous and hence differentiable almost everywhere on $[0, M]$.

Existence of Utility Functions

In¹⁰, it is shown that there are necessary and sufficient conditions for the existence of upper semi-continuous utility functions on arbitrary domains. Preferences are defined on an arbitrary set X in terms of a binary relation \succsim , which is defined as "weakly preferred to". Completeness and transitivity is assumed. For $x, y \in X$, with $x \prec y$, the open interval of alternatives better than x but worse than y is denoted as

$$(x, y) = \{z \in X : x \prec z \prec y\} \tag{16}$$

A preference relation, \succsim is a function $u : X \rightarrow \mathbb{R}$ if

$$\forall x, y \in X : \begin{cases} x \sim y, & \Rightarrow u(x) = u(y) \\ x \succ y, & \Rightarrow u(x) > u(y) \end{cases} \tag{17}$$

This defines a utility function. In terms of existence, a complete, transitive binary relation \succsim on a set X can be represented by a utility function if and only if it is *Jaffray order separable*, which is a way of saying that there is a countable set $D \subseteq X$, such that for all $x, y \in X$:

$$x \succ y \Rightarrow \exists d, d' \in D : x \succsim d \succ d' \succsim y \tag{18}$$

Jaffray order separable automatically works if the domain X itself is countable. In the definition, the set D is countable, as in $\exists n : D \rightarrow \mathbb{N}$ is an injection. In order to find a utility function on the set D , each element $d \in D$ is given a positive weight, with the constraint that the sum of the weights is finite, and the utility of d is defined to be the total weights of the elements that are weakly worse than d . As an example in¹⁰, the weight of $\frac{1}{2}$ is given to the alternative d , with label $n(d) = 1$, weight $\frac{1}{4}$ to the alternative d with label $n(d) = 2$ and so forth i.e. weight $w(d) = 2^{-k}$ to the alternative d with label $n(d) = k$. So if $\sum_{k=1}^{\infty} (\epsilon_k)$ represents a summation of a sequence of positive weights, one can assume, without loss of generality, that the sum is 1. Assign to each $d \in D$ the weight $w(d) = \epsilon_{n(d)}$. Finally, in order to satisfy the preference relation, we define $u_0 : D \rightarrow \mathbb{R}$, for each $d \in D$ by $u_0(d) = \sum_{d' \succsim d} w(d')$. An extension from D to X can be done as

follows, following¹⁰. Let \mathcal{W} be the collection of subsets defined as $\{W(d) : d \in D\} \cup \{\phi, X\}$, and define $\rho : \mathcal{W} \rightarrow [0, 1]$ as:

$$\rho(W(d)) = \sum_{d' \in D: d' \succsim d} w(d') \tag{19}$$

with $\rho(\phi) = 0, \rho(X) = 1$, with the equation above defined for $d \in D$. The set \mathcal{W} is countable, and a covering of X . At this juncture, ρ is extended to an outer measure μ^* on X : for each set $A \subseteq X$, $\mu^*(A)$ is defined to be the smallest total size of sets in \mathcal{W} covering A . A countable collection $\{W_i\}$ of sets W_i from \mathcal{W} covers A if $A \subseteq \cup_i W_i$. From here:

$$\mu^*(A) = \inf \sum_i \rho(W_i) \tag{20}$$

Here, the infimum is taken over all the countable collections $\{W_i\}$ that cover A . Finally, we define $u : X \rightarrow \mathbb{R}$ for each $x \in X$ as the outer measure of the set of elements worse than x :

$$u(x) = \mu^*(W(x)) \tag{21}$$

This leads to the utility representation. With a complete, transitive, Jaffray order separable binary relation \succsim , on an arbitrary set X , the function defined in the previous equation is a utility function for \succsim . It can be shown easily that u represents \succsim .

The Situation

Emotional conflicts are very commonly observed in social media comment sections. These conflicts often take the shape of a clash between two political ideologies but are nevertheless emotional. These conflicts, which we call 'comment wars,' often leave both sides emotionally injured and hateful toward the opposing viewpoint. Hence, we seek to construct a game theoretical model of such comment wars and will propose a solution that a rational and disciplined agent can use to resolve a comment war, allowing both parties to leave in peace without the lasting feeling of bitterness towards the opposition.

Setting Up the Model

We begin the construction of our model by outlining its specific features from a sociological and psychological point of view.

The utility we define in our model is the self-satisfaction of victory along with the public perception that one is winning the argument, which we summarize as "dominance." Naturally, the cost we define is the feeling of defeat and the embarrassment that comes along with it.

We define two players in this game. The first is a "selfish" player who initiates the argument looking for a fight, and is motivated by a desire to maximize one's utility by dominating the argument.

The second player we define is a "disciplined" player who is willing to lose dominance for the greater good, which is cooperation. This player is calm and unemotional - they are committed to following a strategy that encourages cooperation. We define a cooperative relationship as one where both sides concede, producing a positive atmosphere. Fundamentally, it cannot involve one side dominating the other and thus does not require the disciplined player to incur major costs.

Each player, or participant in a social media conflict, is defined to have three possible choices. They can (1) Argue by attacking the opposition, (2) Abstain by making a neutral comment, and (3) Concede by admitting their mistake.

Every new comment posted in a conflictual comment thread is a new choice made. Thus, we interpret this situation as an iterated dilemma, in which a new situation is opened up before a player for them to decide their choice.

The resultant change in total utility for each player is dependent on the reactions of the opponent. For instance, a player who offers peace by conceding who is met with even further aggression will only look like they are being dominated. On the contrary, a player's concession met with another concession will result in a peaceful resolution of conflict where both sides gain some utility. Thus, a prediction of the opponent's most likely response is necessary.

The expected utility or expected reward itself will evolve for every iteration of the game, and will have to be considered in decision-making. For instance, varying levels of confidence in one's ability to win another debate against the opponent will determine the expected reward for attempting to argue once again.

Hence, the two values - a player's predictions of the opponent's most likely response and a player's expected reward from choosing to argue will have to be based on their memory of the conflict so far.

The Prisoner's Dilemma as a Model

The Prisoner's Dilemma, which models two individuals who can either cooperate or defect, offers a very compelling construct that can be applied to conflicts on social media. According to¹¹, a Symmetric 2x2 Prisoner's Dilemma With Ordinal Payoffs take the following form:

	C	D
C	R,R	S,T
D	T,S	P,P

In the above payoff matrix, C indicates cooperation and D indicates defection. The utility values R, S, T, P follow $T > R > P > S$, meaning that while both players defecting on each other leads to a poor outcome for both, and collaboration leads to a positive outcome for both, a player who defects on a collaborative player will gain significant utility.

The inequality of utility values associated with the prisoner's dilemma can also be applied to social media conflicts. Two users who decide to argue will incur the cost of stress and lost time and energy, resulting in dissatisfaction and low utility. Two users who concede to each other will avoid that stress, resulting in higher utility. Yet, if a user collaborates but their opponent argues, they may appear to lose the argument, which can be a source of embarrassment or anger (dis-utility), whereas their opponent will be satisfied by appearing to dominate the argument (extremely high utility).

	Concede	Argue
Concede	6,6	-2,10
Argue	10,-2	2,2

In our scenario, however, the disciplined player is committed to achieving a peaceful resolution of conflict. This player does not seek to win the argument and is unembarrassed by losing the argument, as long as it helps the two walk away without hatred for one another. This player's main source of satisfaction and utility is the knowledge that the level of polarization caused by this conflict is minimized.

Hence, we construct a new matrix using updated payoff values:

	Concede	Argue
Concede	10,6	0,10
Argue	6,-2	-10,2

	Concede	Argue
Concede	R_c, R_r	S_c, T_r
Argue	T_c, S_r	P_c, P_r

Yet, the above construct is not a prisoner's dilemma. While it may seem like an asymmetric prisoner's dilemma, it fails the criteria presented in¹¹. This new matrix does not follow $T_i > R_i > P_i > S_i$

when $i = r, c$, where r refers to the selfish player in the row and c refers to the disciplined player in the column. Yet,¹¹ does provide an alternative set of criteria that preserves the force of the dilemma, which are:

1. $T_r > R_r$ and $P_r > S_r$
2. $T_c > R_c$ and $P_c > S_c$
3. $R_r > P_r$ and $R_c > P_c$

Even here, the matrix fails the second criterion. While¹¹ also claims that one criterion may fail if both players are aware of each other's rationality and payoff matrix, that is not the case in our scenario. While the disciplined player is aware of the payoff matrix of their selfish opponent, the selfish player is unaware of their opponent's discipline and valuation of a peaceful resolution of conflict. Hence, this scenario is not modeled by a prisoner's dilemma.

2 Putting a Preference Structure on the Coordination Process in a Conflict

We use the setup of¹². A Graph Model for Conflict Resolution (GMCR) is used in this regard. A conflict model has a set of Decision Makers (DM), N , a set of feasible states S , a set of relative preference relationships among the states $P = \{P_i, i \in N\}$. Here P_i indicates DM i 's preferences, and a set of directed graphs $G = \{G_i, i \in N\}$. G_i for DM i keeps track of the possible movements in one step from each state. Preferences in the graph model are expressed with a pair of binary relations $\{\succ_i, \sim_i\}$ on the set S , where if $s_1 \succ_i s_2$ then the DM i prefers s_1 to s_2 , and if $s_1 \sim_i s_2$ then DM i is indifferent between s_1 and s_2 . In this regard, we have three states of the set S :

- $\Phi_i^+(s) = \{s_m : s_m \succ_i s\}$; all states that DM i prefers to s
- $\Phi_i^-(s) = \{s_m : s \succ_i s_m\}$; all states that DM i prefers less than state s
- $\Phi_i^=(s) = \{s_m : s_m \sim_i s\}$; all states indifferent to DM i

In this scenario,¹² defines the reachable list $R_i(s)$ from a given state s to be those states that contain all states that DM i can move to in one step. Here, $R_i(s)$ can be partitioned into three subsets as follows:

- $R_i^+(s) = R_i(s) \cap \Phi_i^+(s)$: the set of unilateral improvements (UI) from state s for DM i
- $R_i^-(s) = R_i(s) \cap \Phi_i^-(s)$: all unilateral disimprovements from state s for DM i
- $R_i^=(s) = R_i(s) \cap \Phi_i^=(s)$: all equally preferred states reachable from state s by DM i .

Denoting two DMs by i and j , two types of stability are defined as follows:

- **Nash Stability** Let $i \in N$. State $s \in S$ is Nash stable for DM i if and only if $R_i^+(s) = \emptyset$. Nash stability implies that the state s is stable if and only if DM i can not move unilaterally from state s to any state it prefers more
- **Sequential Stability** For $i, j \in N$, a state $s \in S$ is sequentially stable for DM i if and only if for every $s_1 \in R_i^+(s)$ there is at least one stable state $s_2 \in R_j^+(s_1)$, such that $s \succ_i s_2$. In other words, if a state is sequentially stable for DM i if they do not want to move away from that state through any unilateral improvement, as doing so may result in a unilateral improvement from the opponent that causes the state to be less desirable.

Mathematical Structure of Conflict

We use the structure in¹³. The authors define conflict in a very specific way, stating that the presence of two or more mutually antagonistic impulses or motives is not a conflict. A conflict arises when there is a difference between what a person wants and what he currently has. We set W as the goal and H as the utility a person currently has. The driving force of the conflict is the discrepancy:

$$u = \begin{cases} [W - H], & \text{if goal is } H \geq W \\ [H - W], & \text{if goal is } H \leq W \end{cases} \quad (22)$$

Here $[x] = x$ if $x \geq 0$ and $[x] = 0$ if $x < 0$. In a conflict scenario, there will be multiple conflict parameters that must be considered, and in particular, each person will have different W and H parameters. So any conflict includes a set $\{W_i, H_i\}$ There is a form of discrepancy that has to be resolved when a conflict arises.

Let us assume a theoretical online conflict between a disciplined (peace-seeking) player and a selfish (dominance-seeking) player. In this conflict, if peace and civility have been established, the disciplined player has been satisfied. Even though the selfish player's conflict has not yet been resolved, as their attempts to increase their dominance would occur through civil means (e.g., logic and evidence) which continue to satisfy the disciplined player, the disciplined player will be willing to concede and increase the selfish player's dominance, resolving both conflicts.

Fig. 1 Structure of Conflict

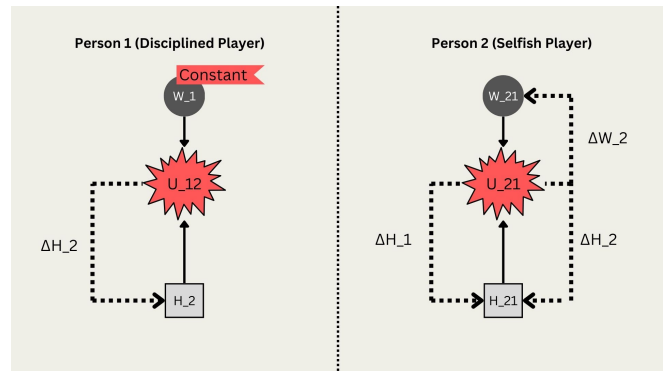


Figure 1 illustrates a conflict control scheme between two users on a social media platform. W_1 indicates the disciplined player's (person one's) desired level of peace or aggression in this argument, while H_2 represents the current level of peace or aggression in this argument. $U_{12} = W_1 - H_2$, meaning that the conflict experienced by person one is that there is not a sufficient level of peaceful dialogue in this argument. Here, it is important to differentiate aggression and conflict. Aggression is the use of force to win the argument (e.g., a political debate) and suppress

the opponent, which may include the use of mockery or strong adjectives. Conflict is simply a difference in wants and haves, separate from the actual topic of the conversation.

In a similar sense, W_{21} represents the selfish player's (person two's) desired level of dominance in this argument, while H_{21} represents the level of dominance they currently have. The conflict is $U_{21} = W_{21} - H_{21}$, meaning player two currently does not have enough dominance.

Let us begin with person one's conflict. We assume person one's desired level of peace W_1 is constant — they always seek to turn this aggressive online argument into a completely peaceful conversation that solely employs persuasion, evidence, and logic. Under the assumption that the disciplined player is able to control their aggression as needed, H_2 , the current level of aggression in this argument, is determined by whether the selfish player (person two) is able to communicate with respect and peace. This influence is represented by ΔH_2 .

Person two's conflict is more complicated. Person two's desired level of dominance W_1 can only be directly influenced by their own values. Hence, we only consider ΔW_2 , person two's impact on their own desired level of dominance. We assume that the effects of their opponent's behavior on their desired level of dominance are still ultimately interpreted by person two themselves and thus incorporated into ΔW_2 . We assume that the current level of dominance person two has is impacted directly by both person two and one—for instance, even though person two may employ strong, aggressive sentences, if person one effectively refutes the arguments with evidence or does not appear suppressed in any manner, person two will be unable to dominate the online argument.

This structure allows for a more realistic depiction of conflict, where multiple parties may not directly be competing for the same resource but will still be bargaining with each other for H values that their respective opponents have complete or some control over. For instance, in the pursuit of dominance, person two may employ intimidating, aggressive language, which would influence person one's conflict (level of peaceful dialogue).

From¹³, one defines, for a participant i , a function that shows the level of conflict i.e. dissatisfaction as

$$U_i = \mu_{i1}u_{i1} + \mu_{i2}u_{i2}, i = 1, 2 \quad (23)$$

where the term μ_{ij} is the coefficient of significance for conflict j for participant i . For rational participants, $\mu_{ij} > 0$ and a conflict is considered resolved when $U_i = 0$ for the first time. In other words, the individual is using W and H as tools to minimize this U function. In a way this is a convex optimization problem, where there is a so-called dissatisfaction function

$$U = b_1U_1 + b_2U_2 \quad (24)$$

with constraints $b_1, b_2 \geq 0$ and normalized to $b_1 + b_2 = 1$. As

an example in question, as in¹³, suppose one has a conflict between a worker and their employer. The conflict exists because the worker wants to get paid more, and the employer wants to improve the quality of the work. In other words, there is a difference between what each has, and what each wants. From the worker's point of view, $u_1 = u_{12} = [W_{12} - H_2]$. Here H_2 is the actual salary, and W_{12} is how much the worker wants to be paid. Now from an employer point of view, $u_2 = u_{21} = [W_{21} - H_1]$. Here, the employer is concerned with the quality of work, so that W_{21} is the desired quality of work, and H_1 is the current actual quality of work. In this scenario, at every interaction, H_1 and H_2 can change to the values $H_1 + \Delta H_1$ and $H_2 + \Delta H_2$. The ΔH_i represents changes in the H_i values. If we assume that a conflict must be resolved, then the u_1 and u_2 values must decrease at each iteration, so it is reasonable that:

$$u_1^{(n)} = [u_1^{(n-1)} - \Delta H_2] \quad (25)$$

and similarly,

$$u_2^{(n)} = [u_2^{(n-1)} - \Delta H_1] \quad (26)$$

The above equations do not require u_i to be a continuous and decreasing variable. For instance, a decrease in peace level, meaning $\Delta H_2 < 0$ would increase $u_1^{(n)}$, the conflict level for the disciplined player.

This is saying that the worker's dissatisfaction is decreasing by the change in H_2 , and the employer's dissatisfaction changes by the change in H_1 . It is also observed that ΔH_i is a function of both u_1 and u_2 . In other words, the change in the worker's real salary and the change in the actual quality of work depends on the excess salary desired and the extra level that the quality of work must improve by. We assume that these are given by functions of the following structure:

$$\Delta H_1 = k_{11}f_1(u_1) + k_{12}f_2(u_2) \quad (27)$$

where f_1 and f_2 are arbitrary functions. Further assumptions on them will be developed.

$$\Delta H_2 = k_{21}f_3(u_1) + k_{22}f_4(u_2) \quad (28)$$

So we can write the expressions for $u_1^{(n)}$ and $u_2^{(n)}$ as follows:

$$u_1^{(n)} = [u_1^{(n-1)} - k_{21}f_3(u_1) - k_{22}f_4(u_2)] \quad (29)$$

and

$$u_2^{(n)} = [u_2^{(n-1)} - k_{11}f_1(u_1) - k_{12}f_2(u_2)] \quad (30)$$

In the above considerations, we assume that k_{ij} are constants, but the functions $f_i, i = 1, 2, 3, 4$ are not necessarily the identity function. The conflict free state implies that $u_1^{(n)} = 0$ and $u_2^{(n)} = 0$ for some iteration n , and it stays at 0 thereafter.

In the simplest case, the functions f_i are all the identity, and in this situation the solution is straightforward. One takes

$$u_1^{(n)} = a\lambda^n, u_2^{(n)} = b\lambda^n \quad (31)$$

In the case the functions are not identities, we use the idea of¹⁴. As a simple instance, suppose we have the relationship

$$u_i^{(n+1)} = P(u_i^{(n)}) \quad (32)$$

Here $P(x)$ is a polynomial of degree m i.e.

$$P(x) = \sum_{k=0}^m a_k x^k, a_m \neq 0 \quad (33)$$

Now suppose the initial starting point of the recursion was $u_i^0 = u_i$, and, following¹⁴, denote by $|u\rangle$ the column vector of the powers of u so that

$$|u\rangle = \{u^j\}_{j=0}^\infty \quad (34)$$

Denoting the vector $\langle e| = [\delta_{j1}]_{j=0}^\infty$, with $\langle e|y\rangle = y$. In any recursion where $u_{n+1} = P(u_n)$ it is true that there will exist a matrix $T = \{T_{jk}\}$ where $u_n = \langle e|T^n|u\rangle$.

In¹⁴, the authors look at the case of linear first-order recursion as equations of the form exactly as in¹³. Starting with the equation for λ as (with the functions f_{ij} being taken as the identity functions):

$$(\lambda - 1)^2 + (k_{12} + k_{21})(\lambda - 1) - k_{11}k_{22} + k_{12}k_{21} = 0 \quad (35)$$

one would get the relationship using the quadratic formula. In order for λ to be real-valued, one needs

$$(k_{11} - k_{21})^2 + 4k_{11}k_{22} \geq 0 \quad (36)$$

It is also assumed that the quantities $[W_{12} - H_2]$ and $[W_{21} - H_1]$ are non-negative. If $\lambda < 1$ it is seen that $u_1^n = 0$ and $u_2^n = 0$ will be stable states.

As

$$\lambda = 1 - \frac{1}{2}(k_{12} + k_{21}) \pm \frac{1}{2}\sqrt{(k_{12} - k_{21})^2 + 4k_{11}k_{22}} \quad (37)$$

So we really need

$$\pm\sqrt{(k_{12} - k_{21})^2 + 4k_{11}k_{22}} < (k_{12} + k_{21}) \quad (38)$$

Simplifying:

$$(k_{12} - k_{21})^2 + 4k_{11}k_{22} < (k_{12} + k_{21})^2 \quad (39)$$

In order to generalize to a non-linear model, we have a fundamental theorem in this regard: Suppose k is a positive integer,

and suppose c_0, c_1, \dots, c_k are constants with $c_0, c_k \neq 0$. Then, the set W of all solutions to the homogeneous linear equation

$$(c_0A^k + c_1A^{k-1} + c_2A^{k-2} + \dots + c_k)f = 0 \quad (40)$$

is a k dimensional subspace of V . This leads to a non-homogeneous operator equation of the form:

$$p(A)f = (c_0A^k + c_1A^{k-1} + c_2A^{k-2} + \dots + c_k)f = g \quad (41)$$

with $c_0, c_k \neq 0$. If W is the subspace of V consisting of all solutions to the corresponding homogeneous equation:

$$p(A)f = (c_0A^k + c_1A^{k-1} + c_2A^{k-2} + \dots + c_k)f = 0 \quad (42)$$

If f_0 is a solution to the $p(A)f = g$ equation, then every solution to that equation has the form $f = f_0 + f_1$, where $f_1 \in W$.

Suppose we have the following advancement operator equation given by

$$p(A)f = (A - r_1)(A - r_2)\dots(A - r_k)f = 0 \quad (43)$$

Here, it is assumed that r_1, r_2, \dots, r_k are distinct non-zero constants. Then, every solution to the previous equation is of the form

$$f(n) = c_1r_1^n + c_2r_2^n + \dots + c_kr_k^n \quad (44)$$

Now suppose that the two equations for ΔH_1 and ΔH_2 are of the form:

$$\Delta H_1(u_1, u_2) = k_{11}(c_n u_1^n + c_{n-1} u_1^{n-1} + \dots + c_1 u_1) + k_{12}(d_n u_1^n + d_{n-1} u_1^{n-1} + \dots + d_1 u_1) \quad (45)$$

$$\Delta H_2(u_1, u_2) = k_{21}(e_n u_1^n + e_{n-1} u_1^{n-1} + \dots + e_1 u_1) + k_{22}(g_n u_1^n + g_{n-1} u_1^{n-1} + \dots + g_1 u_1) \quad (46)$$

We are assuming a polynomial-type relationship here in each of the above equations. Now recall that we had the following structure:

$$u_1^{(n)} = [u_1^{(n-1)} - \Delta H_2] \quad (47)$$

$$u_2^{(n)} = [u_2^{(n-1)} - \Delta H_1] \quad (48)$$

So our new equations in generality become:

$$u_1^{(n)} = [u_1^{(n-1)} - (k_{21}(e_n u_1^n + e_{n-1} u_1^{n-1} + \dots + e_1 u_1) + k_{22}(g_n u_1^n + g_{n-1} u_1^{n-1} + \dots + g_1 u_1))] \quad (49)$$

$$u_2^{(n)} = [u_2^{(n-1)} - (k_{11}(c_n u_1^n + c_{n-1} u_1^{n-1} + \dots + c_1 u_1) + k_{12}(d_n u_1^n + d_{n-1} u_1^{n-1} + \dots + d_1 u_1))] \quad (50)$$

We know that conflict resolution equates to $u_1^{(n)} = 0, u_2^{(n)} = 0$ for some $n > 0$. In the simple case we looked at earlier, if $r \neq 0$ and f is a solution to the operator equation $(A - r)f = 0$, then if $c = f(0)$ then $f(n) = cr^n \forall n \in \mathbb{Z}$.

We can cast the two earlier equations as: (with $u_1^n = 0, u_2^n = 0$)

$$[u_2^{n-1} - k_{11}(c_{n-1}u_1^{n-1} + \dots + c_1u_1) + k_{12}(d_{n-1}u_1^{n-1} + \dots + d_1u_1)] = 0 \quad (51)$$

$$[u_1^{(n-1)} - (k_{21}(e_nu_1^n + e_{n-1}u_1^{n-1} + \dots + e_1u_1) + k_{22}(g_{n-1}u_1^{n-1} + \dots + g_1u_1))] = 0 \quad (52)$$

In the simplest case, one could take $u_1^n = a\lambda^n, u_2^n = b\lambda^n$. However, in the general scenario, if $p(A)f = (A - \lambda_1)(A - \lambda_2)\dots(A - \lambda_k)f = 0$, with r_1, r_2, \dots, r_k are distinct non-zero constants, then $f(n) = c_1\lambda_1^n + c_2\lambda_2^n + \dots + c_k\lambda_k^n$. Suppose, just to increase complexity:

$$\Delta H_1(u_1, u_2) = p_1u_1^2 + q_1u_1 + r_1u_2^2 + s_1u_2 \quad (53)$$

$$\Delta H_2(u_1, u_2) = p_2u_1^2 + q_2u_1 + r_2u_2^2 + s_2u_2 \quad (54)$$

Then

$$u_1^n = u_1^{n-1} - (p_2u_1^2 + q_2u_1 + r_2u_2^2 + s_2u_2) \quad (55)$$

$$u_2^n = u_2^{n-1} - (p_1u_1^2 + q_1u_1 + r_1u_2^2 + s_1u_2) \quad (56)$$

Since we need $u_1^n = 0$ and $u_2^n = 0$, we therefore should have:

$$u_1^{n-1} = (p_2u_1^2 + q_2u_1 + r_2u_2^2 + s_2u_2) \quad (57)$$

and

$$u_2^{n-1} = (p_1u_1^2 + q_1u_1 + r_1u_2^2 + s_1u_2) \quad (58)$$

Since there are no general methods to solve this, we will assume that it is possible to reduce the above equations into the following form:

$$u_1^{n+1} = f_1(n)u_1^n + g_1(n)u_1^{n-1} \quad (59)$$

$$u_2^{n+1} = f_2(n)u_2^n + g_2(n)u_2^{n-1} \quad (60)$$

Following the method of ¹⁵, we define an auxiliary sequence $\{b_n\}_{n=1}^\infty$, by (this works for both u_1 and u_2):

$$b_n = u_{n+1} - \lambda(n)u_n, n \geq 1 \quad (61)$$

In addition, set

$$u_{n+1} - \lambda(n)u_n = \mu(n)(u_n - \lambda(n-1)u_{n-1}), n \geq 2 \quad (62)$$

From here

$$u_{n+1} = (\lambda(n) + \mu(n))u_n - \mu(n)\lambda(n-1)u_{n-1} \quad (63)$$

Then comparing coefficients:

$$f(n) = \lambda_n + \mu(n) \quad (64)$$

$$g(n) = -\lambda(n-1)\mu(n) \quad (65)$$

It can be shown that for any pair of f, g the λ, μ always exists. So, for instance, if

$$u_{n+1} = \lambda(n)u_n + b_n \quad (66)$$

and

$$b_n = \mu(n)b_{n-1} \quad (67)$$

then

$$u_{n+1} = b_1 \prod_{i=2}^n \mu(i) \left(1 + \sum_{k=3}^n \prod_{i=k}^n \frac{\lambda(i)}{\mu(i)} + \frac{u_2}{b_1} \prod_{i=2}^n \frac{\lambda(i)}{\mu(i)} \right) \quad (68)$$

In other words, in this case, conflict resolution occurs only when

$$b_1 \prod_{i=2}^n \mu(i) \left(1 + \sum_{k=3}^n \prod_{i=k}^n \frac{\lambda(i)}{\mu(i)} + \frac{u_2}{b_1} \prod_{i=2}^n \frac{\lambda(i)}{\mu(i)} \right) = 0 \quad (69)$$

for some finite integer value of n .

In terms of a functional relationship, in the simplest case where equations 9.8 and 9.9 hold true, we need equation 9.18 to hold true if a conflict can at all be resolved. In a more general case, we need equation 9.48 to hold for both u_1 and u_2 , assuming that equations 9.38 and 9.39 show conflict evolution.

Applications to Online Arguments

Let us return to the idea of conflict in social media comment sections, to analyze how the mathematical conclusions of this paper can be applied. We will analyze the situation from the perspective of the disciplined player, who seeks to de-escalate the conflict and encourage peaceful dialogue.

In the first linear case, conflict is resolved, meaning $u_i \rightarrow 0$, when equation 9.18 is satisfied. This equation essentially suggests that k_{ij} must remain approximately similar. In real-life terms, both players' satisfaction must evolve at a similar rate to ensure resolution, meaning when the disciplined player achieves some peace in their argument, the selfish player must also gain a sense of progress (i.e., dominance). This offers some guidance on a conflict resolution strategy a disciplined player can take to encourage conflict resolution: assuming the selfish player is becoming more peaceful, they must reward it by partially 'losing' the argument.

The second case, following a nonlinear structure, is more nuanced. We have concluded that conflict resolution will only occur when $\mu(i) = 0$. μ is the "additional" dissatisfaction added to each player's u , increasing the gap between what they desire

and what they have. From the perspective of a disciplined player in an online conflict, new is the additional aggression added to the conflict by the selfish player. Thus, to make $\mu = 0$, the disciplined player must create a situation where the selfish player does not become more angry. Equation 9.47 shows that this situation only needs to be achieved once to guarantee conflict resolution.

Next Steps

Accurately modeling a complex social interaction occurring over the internet involves multiple additional layers of complexity. Thus, we plan to enhance our current model by incorporating the following additional considerations.

Stochastic Components of Conflict Resolution

Let's denote the parties involved in the conflict by A and B . We also assume that the entire conflict resolution process can be modeled as a game where each party makes decisions or acts in a manner so that some form of respective payoff is maximized. This leads to a stochastic decision process. Suppose x_A and x_B represents the actions of parties A and B , and suppose the payoffs are $u_A(x_A, x_B)$ and $u_B(x_A, x_B)$. Now represent the stochastic component by ξ which in turn affects the payoffs. In this we write the stochastic payoffs as:

$$U_A(x_A, x_B, \xi) = u_A(x_A, x_B) + \varepsilon_A(\xi) \tag{70}$$

$$U_B(x_A, x_B, \xi) = u_B(x_A, x_B) + \varepsilon_B(\xi) \tag{71}$$

Here $\varepsilon_A(\xi)$ and $\varepsilon_B(\xi)$ are random variables that represent the stochastic effects on the payoffs. One could also assume, in a simple case, that the following distribution structure holds:

$$\varepsilon_A(\xi) \sim N(0, \sigma_A^2) \tag{72}$$

$$\varepsilon_B(\xi) \sim N(0, \sigma_B^2) \tag{73}$$

The variances in the above are indicative of the uncertainty here. The expected payoff in such a conflict scenario can be written as

$$E[U_A(x_A, x_B)] = u_A(x_A, x_B) \tag{74}$$

and similarly

$$E[U_B(x_A, x_B)] = u_B(x_A, x_B) \tag{75}$$

where we understand that $E[\varepsilon_A(\xi)] = E[\varepsilon_B(\xi)] = 0$

In this scenario, the actions of the individuals A and B depend on what risk levels they are willing to undertake. As an example, if A happens to be risk averse, they may choose to maximize a

utility function that punishes the variance component. One form such a function could take would be:

$$V_A(U_A) = E[U_A] - \frac{\lambda_A}{2} Var(U_A) \tag{76}$$

The risk aversion coefficient is represented by the variable λ_A . In this setup, the idea of Nash equilibrium as a type of stochastic equilibrium emerges. This means that both parties would choose a course of action that maximizes their individual expected utility given the actions of the other party. This is represented as:

$$x_A^*, x_B^* = argmax_{x_A, x_B} E[V_A(U_A)], E[V_B(U_B)] \tag{77}$$

It is seen that the equilibrium incorporates the uncertainty. In particular, this will give rise to a Stochastic Differential Equation setup. Since the conflict resolution scenario evolves, one could, for instance, write these as:

$$dx_A(t) = f_A(x_A, x_B, t)dt + \sigma_A(x_A, x_B, t)dW_A(t) \tag{78}$$

$$dx_B(t) = f_B(x_A, x_B, t)dt + \sigma_B(x_A, x_B, t)dW_B(t) \tag{79}$$

The $W_A(t)$ and $W_B(t)$ are Wiener processes that represent the stochastic component, and the terms $\sigma_A(x_A, x_B, t)$, $\sigma_B(x_A, x_B, t)$ represent volatility terms. The outcomes are not deterministic; they depend on random variables which influence the decisions and payoffs of the involved parties. Let's understand the terms in the above equations. The variables $x_A(t)$, $x_B(t)$ represent the actions of A and B and these change with time t . The terms f_A, f_B are called drift terms and represent the deterministic part of the dynamical structure. These are strategies that exist when no randomness is present. The σ_A, σ_B represent what are called diffusion terms, and these account for the randomness in strategy evolution. Finally, we have $W_A(t)$, $W_B(t)$ which are called Wiener processes, or Brownian motions, which are independent of each other and represent the source of the randomness. In this specific structure, the following properties would hold:

1. $W_A(0) = 0, W_B(0) = 0$, so the motions start at 0
2. $W_A(t)$ and $W_B(t)$ have independent increments so that $W_A(t + \delta t) - W_A(t) \sim N(0, \delta t)$
3. The paths of $W_A(t)$ and $W_B(t)$ are continuous but nowhere differentiable

The drift term gives an average rate of change of x_A, x_B over time, and can represent processes like learning, adaptation or strategic changes. It is entirely possible that the strategies of the two parties are interdependent and these are usually represented in the equations. Let's look at a very simple example. Suppose the individuals are negotiating, and their willingness to compromise is represented by $x_A(t)$ and $x_B(t)$. Now suppose $f_A(x_A, x_B, t) = \alpha_A(x_B - x_A)$ represents the notion that A is more likely to move

towards B 's ideas if $x_B > x_A$. Now suppose $\sigma_A(x_A, x_B, t) = \beta_A$ is a constant representing the uncertainty in A 's strategy. Then the Stochastic Differential Equation for $x_A(t)$ would be:

$$dx_A(t) = \alpha_A(X_B(t) - x_A(t))dt + \beta_A dW_A(t) \quad (80)$$

Solving such an equation would show how A 's strategy would evolve over time.

Conclusion

In this paper, we have proposed a conflict resolution strategy under two frameworks: a structure where individual satisfaction changes linearly, and a structure where individual satisfaction changes nonlinearly. We have proven that specific criteria must be met to guarantee conflict resolution (i.e., a situation where one individual is satisfied) and applied it to the context of social media comment section arguments. In particular, we have formulated a potentially effective conflict resolution strategy for a disciplined user seeking to achieve peaceful dialogue in a heated online argument.

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