Analysis of Geometric Representations used to Simplify Spacetime Curvature of Black Holes

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Received April 23, 2024
Accepted August 07, 2024
Electronic access September 15, 2024

Black holes remain partially unexplained due to incomplete understanding of their functions. Theoretical approaches often rely on complex mathematical formulas, which can be inaccessible to new learners lacking advanced mathematical skills. To bridge this gap, diagrams such as Penrose, Kruskal-Szekeres, and Eddington-Finkelstein, used in combination with mathematics, can help explain these concepts more effectively. However, the most efficient diagram for this purpose has not been identified, requiring learners to determine which works best for them. A literature review analyzed the strengths and weaknesses of each diagram in teaching environments, concluding that the Eddington-Finkelstein diagram is most effective for new learners. The review first introduced diagrams used to show spacetime before focusing on those that specifically show the spacetime curvature of black holes. The review included papers on how each diagram displayed a black hole's curvature of spacetime, to analyze if they reflected concepts in General Relativity, Special Relativity, and black hole physics. It also included articles and user studies on the use of diagrams and visual representations for teaching physics concepts, to determine if the diagrams discussed would be suitable for teaching black hole physics. The future of diagrams was also looked at, considering the rise of computer simulations and tools that can automate the manipulation of diagrams and how they function in a classroom environment. Eventually, the development of diagrams will help bridge the knowledge gap, in theoretical and still unexplained physics like black holes, between those researching in the field to those who are just beginning their journey.

Introduction

A common approach for theoretically understanding the effect of black holes on their surroundings is using only complex mathematical formulas and metrics. This key understanding is locked off from new learners who do not have enough experience with higher level mathematics. In a study with 45 undergraduate students, most of them struggled to correctly apply equations to concepts in relativity, especially since they were dealing with events that are not commonly observed [1]. In addition, another study showed that very little students were able to correctly understand introductory General Relativity, a key concept for understanding black holes, undermining the importance of another teaching method [2]. Both studies used traditional learning practices, relying just on mathematical formulas, which is why we analyzed a different way to establish solid foundations on the understanding of black holes.

Rather than attempting to explain complicated concepts with just mathematics and traditional teaching, another approach can be used: visual representation. Visual representations are a way to communicate phenomena that is not commonly observed and can be used as a new educational approach [1]. They have been used in science for centuries, even within ancient civilizations, and are an effective method of building on scientific knowledge and theory [3]. It is often difficult to visualize and understand physics when using abstract concepts, but visual diagrams have been proven to enable a solid conceptual understanding [4]. A study analyzing how General Relativity is taught, specifically in secondary schools, concluded that the educational model relied heavily on mathematics instead of focusing on conceptual understanding, as the researchers suggested [5]. In another study on teaching General Relativity, all the experimental groups taught with a new educational approach, that used visualizations, significantly outperformed the control groups taught with a traditional approach [6]. The participants were all non-physics major students, proving that an interactive and visualization reliant approach is a more effective way than traditional learning to communicate complex topics [6].

Using interactive models can explain concepts to a wider audience, but models must be used to preserve the true meaning of ideas while also keeping them simplified. To explore and determine what models best maintain this balance, we analyzed geometric representations used to simplify an understanding of the spacetime curvature of black holes. To understand how black holes interact with their surroundings, an understanding of related concepts must be established.

An understanding of general relativity is necessary, as black holes are basically predicted with the theory. General relativity is used to explain the relation between mass, spacetime, and the force of gravity: mass bends spacetime, three-dimensional space with a dimension of time, and that creates what we perceive as gravity. Black holes can be understood as obeying the same rules, just taking it to the extreme as they are extremely massive but condensed in a small area, like a dumbbell compared to a beach ball. However, just that analogy, or any written explanation is not the best way to describe black holes.

The Speed of Light and Special Relativity

The classical physics used before the 20th century heavily relied on human intuition and attempted to explain events that happened in everyday life[7]. People believed that the understanding of physics was complete, and because technology was not advanced enough to perform experiments at high velocities, much of the astronomical phenomena remained unexplained. However, Einstein changed everything by introducing his view of spacetime and gravity, which directly challenged the principles of Newtonian physics, the dominant explanation for the universe. Physicists had never been able to put Newtonian physics to the test in all areas, specifically high velocities near the speed of light, and had a flawed perception of a complete understanding[8]. However, Einstein's proposed explanation for the universe was not immediately accepted as the central concept of physics, and his ideas only made an impact later. Contrary to popular belief, Einstein did not actually win the Nobel Prize for his work on relativity but for discovery for the photoelectric effect, showing how unrecognized his other work was, even though it turned out to be groundbreaking[9]. When they did get widely accepted, Einstein's theories revolutionized the way physicists viewed the universe, now as an interconnected spacetime, and gave them the foundational pillars needed to explore previously inconceivable phenomena. Without Einstein, black holes would not have been known to exist, and the era of modern-day physics research would not be the same. Every idea starts from another, and Einstein came about his groundbreaking conclusions through a changed view of spacetime, founded on the results of the Michelson-Morley experiment, which proved classical Newtonian physics could not provide an accurate explanation of high velocities near the speed of light[7].

Before the experiment, there was a clash between Maxwell's proposed electromagnetic laws and Newtonian rules. Maxwell disagreed with Newton's principle of equivalence, stating that the same laws could be observed from any perspective and all observers were equivalent and sought to prove that different observers looked at different laws, turning to the existence of light as support[7]. He proposed that there was a medium called the ether that existed in free space, such as the vacuum of space, to explain that light could not travel through nothing. The experiment involved using an interferometer, where light beams would bounce off mirrors and a half-silvered mirror, where half the light is reflected, and the time for the light to be detected would be measured[8].

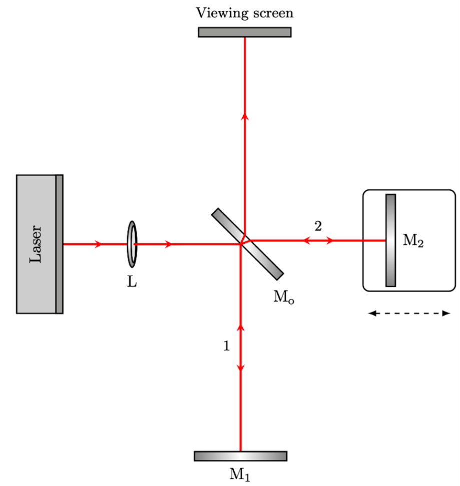

Figure X Interferometer from[10]

Figure X displays the interferometer as used in the experiment. A laser is emitted from the laser box and then travels through the lens at L to the half-silvered mirror, located at Mo, where it then gets split into two beams. One beam will get reflected and travel on path 1 towards the immovable mirror M1, before being reflected and passing through the half-silvered mirror and reaching the viewing screen. The second beam passes through Mo on path 2, gets reflected by the movable mirror M2, and then gets reflected by Mo and directed towards the viewing screen.

The goal was to prove the existence of the ether because the interferometer was moving on the same axis which M2 moves, due to the motion of the Earth, and the beam on path 2 was predicted to travel a different distance. Since the distance differed, the light beams were expected to reach at different times, but the measured time ended up being the same[8]. Regarded as one of the biggest experiment failures in history, it instead proved that the speed of light acts independently of the motion of its source and provided a starting point for Einstein's theory of relativity[7]. Not only is the speed of light constant, but other speeds would also act independently of the source, forcing physicists to rethink how time, distances, mass, and energy worked, essentially the entirety of physics.

One of the founding principles of special relativity is the speed of light being constant, and the other is the principle of equivalence[7,11]. The principle of equivalence, as mentioned previously, states that each inertial observer, meaning no forces are acting on them, sees the same physical laws[7]. The speed of light is important to relativity because it does not just determine the speed at which light moves, but also determines the total speed of all objects in spacetime — a value combining an object's speed through time and space[12,13]. For example, an object with a lower speed or less movement in space has a larger stationary movement through time[12]. The speed of light is one of the main reasons why relativity disagrees with the Newtonian view of time and distances being the same for all observers, as what each observer views is affected by their speed, especially when they are traveling close to the speed of light[14, 11]. Observers see events at different times, such as a clock having velocity appearing to run slower, and view objects with contracted lengths compared to their length at rest, due to them having different perspectives[14, 11]. The closer an object, such as a rocket, travels to the speed of light, the faster the time passes for an observer traveling in the rocket[12]. In addition, objects have a relativistic mass, which is a result of the object traveling at a velocity near the speed of light[12]. The cause and effect of events is also not consistent among observers, called the relativity of simultaneity: two observers do not always view the same events happening at the same time or in the same order[7, 14]. The relativity of simultaneity is vital to understanding how events in spacetime and general relativity work, and helping model spacetime. The mathematical relation of time dilation and length contraction can be seen in the left equation in Figure 1.M. The Lorentz factor, y, is also the time dilation factor and the length contraction factor. The v is the velocity of the object and c is the speed of light. The right equation shows how relativistic mass is related to the rest mass, by the Lorentz factor, where m is the relativistic mass, m_0 is the rest mass, and y is the Lorentz factor.

$$y = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m = y \times m_0$$

Modeling Spacetime and the Basics of General Relativity

Special relativity describes how observers view objects when they are traveling at a certain velocity, but general relativity adds to the description by focusing on the acceleration of objects through the equivalence principle: the force of gravity is indiscernible from the force of acceleration, observers that are accelerating at the same rate are equivalent, and all observers are equivalent if gravity is the only force acting on them[7, 15].

General relativity is used to describe the effect gravity has on the universe, and that understanding can be built on by exploring the fabric of the universe. The space that we live in is commonly known as only three-dimensional, meaning there are three spatial coordinates, but there is a fourth dimension of time, creating a time coordinate[16]. Adding the fourth dimension of time results in a new way of looking at the universe: an interconnected *spacetime*[16]. General relativity explains how gravity is caused by mass warping the spacetime around them, like how a bowling ball dropped on a blanket that is stretched tight will bend around the ball due to its weight[15]. If another object with less mass, such as a light ball, is added to the blanket, it will move towards the more massive bowling ball as expected. This basic example explains how mass warps spacetime around itself and causes other objects to "fall" towards it[17].

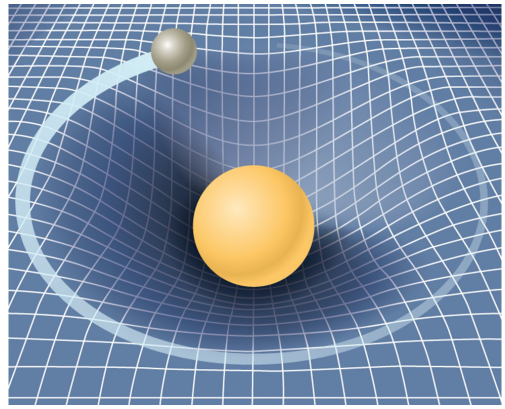

Figure 1.A Blanket Depiction of Gravity in Spacetime from[18]

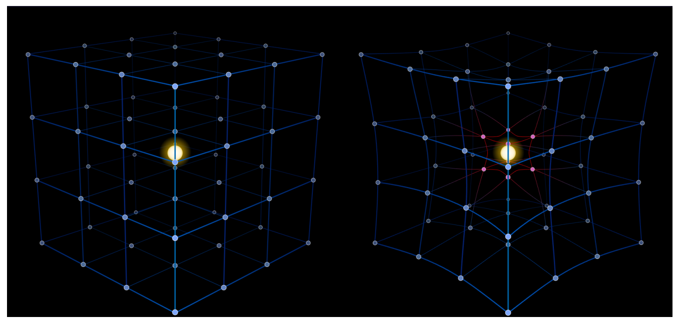

Figure 1.B Three-Dimensional Depiction of Gravity in Spacetime adapted from[19]

However, the blanket depiction in Figure 1.A using general relativity is not the most accurate, as it only shows a two-dimensional depiction as opposed to a three-dimensional view of four-dimensional spacetime, which is not commonly used but shows a better visualization[17]. The three-dimensional depiction shows an object with mass contracting a three-dimensional grid towards itself and showing the motion of objects in curved spacetime, which will be further explored when discussing curvature[17]. As seen in Figure 1.B, the presence of an object with mass distorts the grid of spacetime, and the object pulls its surroundings towards itself. Despite the inaccuracies, the basics of spacetime curvature for visualizing black holes can still be understood by using two-dimensional depictions instead of three or four-dimensional diagrams.

The simplest form of spacetime is without any warping: flat Minkowski spacetime[7]. Since it has no mass, and therefore no curvature, a simple graph can be used to represent it as shown in Figure 1.C. The vertical axis is labeled to show the speed of light multiplied by time, and the horizontal axis is labeled with a spatial coordinate, meaning two spatial coordinates are suppressed. The speed of light, c, multiplying the time, t, allows

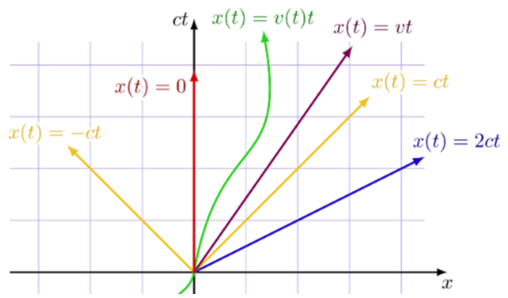

Figure 1.C Flat Minkowski Spacetime Diagram adapted from[21]

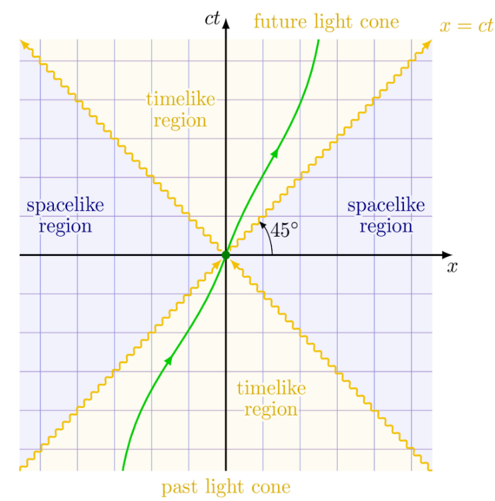

Figure 1.D Light Cone Diagram from[21]

both the vertical and horizontal to have the same units. The different lines on the diagram are called worldlines, and they show an object position as they move through spacetime[16]. For example, a light ray is shown to be traveling with a yellow line in a positive direction, at a 45-degree angle[20]. This is because the graph plots x vs. ct, so the displacement in x will be the same displacement in ct for an object traveling at the speed of light, like a photon. On this diagram, and many others, light only travels at 45 degrees so something with a larger angle will be slower than the speed of light and something with a smaller angle will be faster, which is not possible as discussed earlier[20]. The object with a red worldline remains in the same spot throughout time, hence why it is vertical. The magenta worldline represents an object with a constant velocity, resulting in a straight line. The green worldline represents an object with changing velocity over time, which is why the line is curved. The green, magenta, and red paths are all possible because they are all moving forward in time towards the top of the diagram with an angle, from the position axis to the lines, greater than the path of light. However, the blue worldline is not possible as the object is traveling faster than the speed of light with an angle less than 45 degrees and a speed of 2c.

There are 3 different types of separation of events: time-like, space-like, and light-like[13]. Time-like separated means the events can occur in the same spot but not at the same time, space-like separated means the events can occur at the same time but not in the same spot, light-like separated means the events can be traveled to in a beam of light[13]. The possible past and future of any object can be shown with a light cone, shown in Figure 1.D, which shows the possible paths light could take if emitted from a single point. It still maintains the fact that light travels at 45 degrees to show all the possible futures of the object, all the possible pasts, and the areas that it would be impossible to go to without breaking the speed of light[20]. The jagged yellow line labeled x = ct represents a light ray, together with a ray in the other direction, represent the light cone. As shown in the previous spacetime diagram, objects can travel a path with an angle greater than 45 degrees, which puts them inside the light cone[20]. The green line represents an example path of an object, passing through the origin. The future region shows where the object can travel, which resides inside the light cone. Similarly, the past region shows where the object could have originated from. The time-like and space-like regions are labeled, and the light-like region resides on the jagged yellow lines. The light cone diagram is a great diagram for predicting the possible paths of objects with mass through spacetime and can be used on other diagrams to understand how they affect these objects.

The more complicated version of spacetime is when it has any mass on it because then it results in the curvature of the spacetime and what is observed as gravity. When this curvature of spacetime occurs, objects do not follow a straight line as they would in flat spacetime, but rather have a deflected trajectory or start orbiting a massive body, resulting in a curved path through spacetime, called a geodesic[8,22,23]. A geodesic describes the possible path that can be taken from one point to another on a curved surface, and all objects will naturally try to take the shortest path[22,23]. Even though a straight line is the shortest possible path an object can take on a flat surface, a geodesic is not always the shortest possible path on a curved surface but just the shortest when compared with nearby paths[23].

Unlike a geodesic on a two-dimensional surface, a geodesic in four dimensions refers to the shortest possible path that can be taken through space along with time, through spacetime[23]. By default, any object with mass follows a straight path through time toward the future, but the curvature of spacetime results in a bent trajectory where the object moves in response to a more massive body[8]. On a spacetime graph, such as the Blanket Depiction in Figure 1.A, for example, the path an object takes revolves around the center of gravity, from a massive object, because the fabric of spacetime is twisted around it[8,23]. An object continues moving into the future, but gravity is what results in that path appearing to be curved[8,23]. Any massive body causes other smaller objects to follow geodesics due to gravity, such as the Moon orbiting the Earth, and the behavior of objects near black holes[23]. Their curvature results in altered distances in spacetime along with inevitable events, and objects taking geodesics are a key part of understanding black holes' interactions with spacetime.

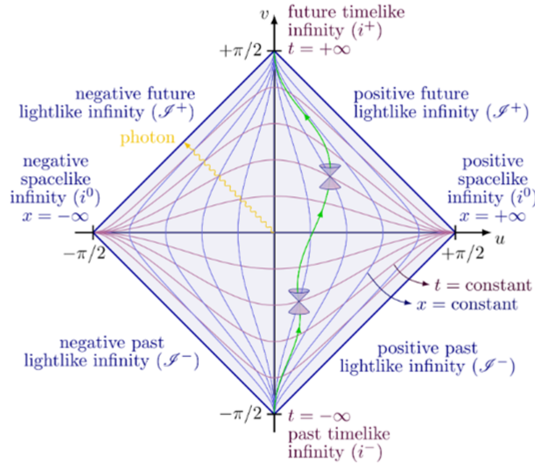

Figure 2.A Penrose Diagram of Flat Spacetime from[21]

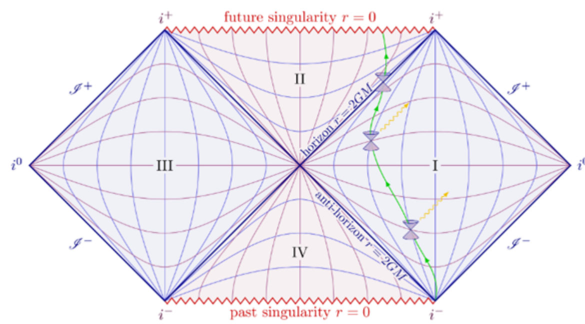

Figure 2.B Penrose Diagram of a Schwarzschild Black Hole from[21]

Modeling Black Holes and their Effect on Spacetime

Before moving on to more complicated diagrams to explain spacetime, the foundations for all the diagrams need to be discussed. When a diagram is created to represent spacetime, they differ because of the different equations that are used as a mathematical basis. The equations for each of the diagrams are used to create coordinate systems, which are numbers or coordinates used to show the position of an object in spacetime. Rather than a typical two-dimensional coordinate system with two numbers, or a three-dimensional coordinate system with three numbers, the coordinate system for spacetime has a fourth coordinate to represent time. The creation of different coordinate systems is a result of addressing issues with previous coordinates, such as a coordinate singularity[7]. A coordinate singularity appears the same as a physical singularity on coordinate systems: an unexplainable point in spacetime that seems to break its foundational equation[7]. New coordinate systems are created to work around coordinate singularities but if it persists in multiple models, then it is a true physical singularity, such as a black hole[7].

Before diving into the way black holes work, their formation and basic structure should be discussed. A black hole, by definition, is a region of spacetime that creates so much curvature that matter, and even something as fast as light, cannot escape from it. A black hole is made up of the event horizon, a nonphysical barrier that marks the point of no return from the black hole, and singularity, which is the infinitely dense point at the center[24]. The singularities of black holes are still unexplained in the way that they interact with spacetime and what they are, but black hole diagrams provide reasonable predictions for them. A black hole is formed when an extremely massive body collapses because the unstoppable force of gravity acting on the massive object, like a very massive star much larger than our Sun, suddenly compresses it, but stops at the black hole's event horizon[24]. The event horizon is dependent on the mass of the black hole and is a certain value for any mass. For example, for the Earth to become a black hole with the mass it has, it would need to be compressed into an object around two centimeters in diameter, meaning the diameter of the black hole would be around two centimeters.

Before attempting to model every type of black hole in the universe, as they are all different, the diagrams should be explored with the most basic black hole, the Schwarzschild black hole. A Schwarzschild black hole is a nonrotating black hole that does not have a charge and has its event horizon bounded by the Schwarzschild radius[24]. It is the simplest form of a black hole and therefore can be easily modeled compared to other black holes, but most likely does not exist[25,26]. Stellar black holes, which are the most common, are formed in the supernova of an extremely massive collapsed star, and the collapse results in momentum that is preserved and maintained by the black hole[24]. In addition, any microscopic particle can make a nonrotating black hole start rotating, making it unlikely for a Schwarzschild black hole to exist[24,25].

To model a black hole in spacetime, the entirety of spacetime needs to be modeled in a finite amount of space, and it can be done with the Penrose diagram, shown with Figure 2.A[27]. A Penrose diagram compresses infinite spacetime into a diamond shape, with the left corner marking infinite positive space, the right with infinite negative space, the top with infinite future, and the bottom with infinite past[28,29,20]. Just like other spacetime diagrams, light travels at strictly 45-degree angles everywhere on the diagram[29,20].

A Penrose diagram for a Schwarzschild black hole resembles four diamonds, so four distinct sections, shown in Figure 2.B. Section one is the observable universe and can be represented on its own with a flat spacetime Penrose diagram. Section three is the parallel universe, but there is no known way of traveling to it[26]. Across sections one and three is a non-traversable wormhole, meaning nothing can pass through it and into the parallel universe unless it travels at the unachievable speed of light[26,27]. The fourth region is a white hole, which behaves like the opposite of a black hole since nothing can enter it and everything inside of it is ejected into the universe or parallel universe by crossing the horizons bounding the parallel universe and universe[25,26,27,28]. However, even though a white hole can be possible, there is a high likelihood that it does not exist. The second region is where the black hole is located, with the singularity represented by the jagged line. In region one, time runs vertically, and space runs horizontally, from left to right, but inside the black hole in region two, they switch[29]. The event horizon is marked by the boundaries of region two, with two lines traveling at 45 degrees labeled with the Schwarzschild radius, r = 2M. Since light rays can only travel at 45-degree angles, they cannot go into the fourth region or leave the second region, as the 45-degree lines used as boundaries can only be crossed with an angle of travel showing faster than light travel, which is not currently possible[30].

A light cone can be used to better understand why not even light can escape the black hole. Imagine an object traveling towards the black hole as it gets pulled towards the event horizon, illustrated with the path of the light cone in Figure 2.B. Before reaching the event horizon, the object's light cone shows possible futures that are outside of the black hole. However, the light cone eventually pokes past the event horizon, lining up its 45-degree boundaries with the boundaries of the event horizon, so all the object's possible futures only lie inside the black hole[31]. The only way to escape would be faster than light travel, which would violate the object's light cone boundaries[31]. Once inside the black hole, the object is doomed to fall towards the singularity. However, the relativity of simultaneity creates inconsistencies in what is observed from someone falling into the black compared to someone looking at them from outside the event horizon[25].

When falling towards the black hole, the clock of an outside observer would appear to run slower than the observer entering the black hole, but an outside observer would see the opposite[32]. To them, the object or person nearing the event horizon would never actually appear to enter the black hole, as all the light from the light cone only goes inside the black hole[31,32]. The object would stay in place on the event horizon for an infinite amount of time. The Penrose diagram can compress all spacetime into a finite area on a diagram, resulting in a representation of entire observable universes in one diagram. However, it does not entirely accurately depict how specific parts, like the singularity, are visualized in four-dimensional spacetime due to only being a two-dimensional representation and creating a simplified but

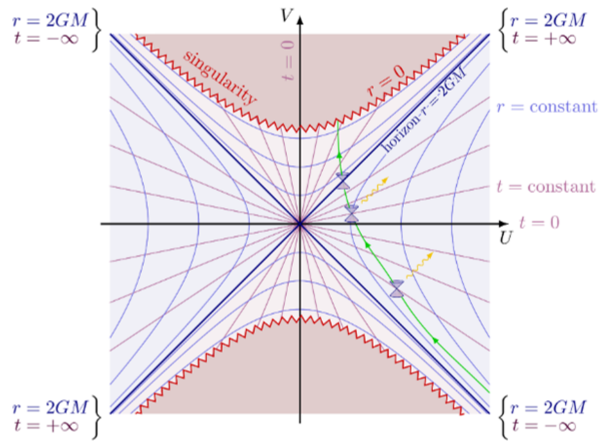

Figure 3 Kruskal-Szekeres Diagram of a Schwarzschild Black Hole from[21]

false visualization.

When using Schwarzschild coordinates to explain black holes, certain values result in coordinate singularities, but using the Kruskal-Szekeres coordinates avoids the coordinate singularities and makes a more accurate diagram, as shown in Figure 3. The Kruskal-Szekeres diagram of spacetime is like a Penrose diagram but has a few noticeable differences[33]. The similarities will first be discussed before moving on to the differences and unique functions of the Kruskal-Szekeres diagram.

Figure 3 uses a time axis that travels vertically, and a spatial axis that travels horizontally, like the way a Penrose diagram is designed, but Figure 3 is a finite amount of space. Region one represents the spacetime of the universe and shows the curvature of spacetime when approaching a black hole's event horizon. Region three is a parallel universe, like region three in the Penrose diagram. The 45-degree lines labeled with r = 2M represent the event horizon of the black hole, so an object from regions one and three can fall into region two, which is the inside of the black hole[34,35]. Any object falling towards the black hole's event horizon will reach at the same time as any other object that had fallen towards it, from any place and at any time[35]. The behavior relates to the relativity of simultaneity, where observers falling toward the black hole at different times would see them both enter at the same time while distant observers would see them both frozen in time at the event horizon[35].

The upper hyperbola shows the singularity of the black hole, which every object in a black hole must approach[34]. Region four is the white hole, which is connected to the third region, which is a mirror replica of the spacetime in region one[34]. Objects with mass can come out of the white hole into region one, region two, and region three. Nothing can enter the white hole, as it will only eject matter and energy[25,26,27,28]. Matter and energy can fall into the black hole in region two, but no information can be transported from region one to region three, or vice versa[26,27]. There is a wormhole at the center like the Penrose diagram, but crossing it requires greater than light speed and therefore is not possible[26,27].

In regions one and three, the distance coordinate goes outward but in regions two and four, the distance coordinate switches with time and goes inward[36]. Just like other spacetime diagrams, light rays travel with an angle of 45 degrees, and objects must have a higher angle than 45 degrees, so they do not break the speed of light[34,36]. When outgoing light rays are released into regions two and four, the light rays in region two will go to the singularity of the black hole but will go from the white hole, region four, into region one[36]. When ingoing light rays are released from region one, they will enter the black hole, region two, and go to the singularity but they cannot enter the white hole, region four, and go into region three[36].

The curved lines in each region in Figure 3 show the contour lines of the coordinates, illustrating the curvature of spacetime and helping predict the path objects with mass will take, a geodesic, as they are affected by the black hole's gravity[37]. Because of the intensity of spacetime curvature black holes cause and the fact that objects must move forward in time, the geodesics of objects will always curve towards the event horizon, drawing them towards the center of gravity: the black hole[38]. The geodesic lines inside of the black hole shows the path an object takes as they inevitably end up at the singularity and pass the event horizon from region one to region two[37]. No geodesics show a path through spacetime that allows an object to escape the event horizon from region two, which is why black holes can trap light and faster than light travel is required to escape them[37].

The curved hyperbolas in the first and third regions show the path of having constant distance and the straight lines in the same region show the path of having constant time[39,36]. Unlike a Penrose diagram, the Kruskal-Szekeres diagram can more accurately depict how event horizons and singularities interact with spacetime and show the curvature of spacetime. The diagrams preserve spacetime geometry with a four-dimensional view and better explain how the different regions are connected by focusing on a finite section of the universe near the black hole. However, because it focuses on a specific section, it is unable to show black holes' different interactions with spacetime far away from the black hole compared to closer at the same time[33].

The Eddington-Finkelstein coordinates also eliminate coordinate singularities, and the corresponding diagram is especially useful in visualizing how matter and energy behave when entering a black hole, with Figures 4.A, 4.B, and 4.C. The y-axis represents time and the x-axis represents space, using r. R = 0 is the location of the black hole's singularity, and r = 2M is the Schwarzschild radius of the black hole, marking the event horizon and point where objects can no longer escape from the black hole's gravitational pull[40]. Traveling to the

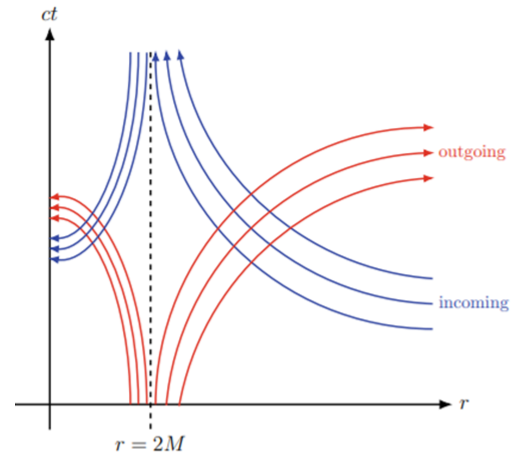

Figure 4.A Eddington-Finkelstein Diagram of a Black Hole Distantly Observed from[7]

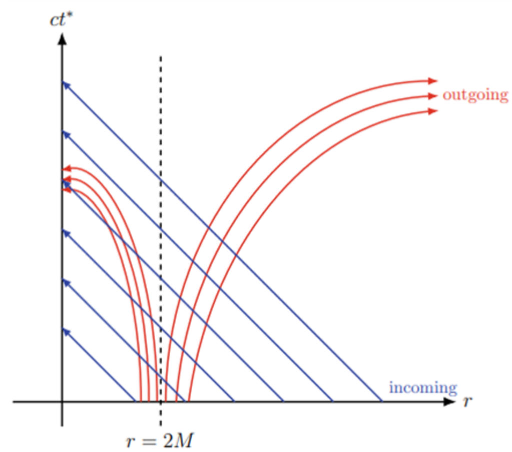

Figure 4.B Eddington-Finkelstein Diagram of a Black Hole observed near Event Horizon from[7]

left moves away from the black hole and traveling upwards is moving forward in time[40]. The lines with arrows in the diagrams represent the geodesics light rays will take when near the Schwarzschild radius of a Schwarzschild black hole, showing the paths that objects will take[7].

Figure 4.A shows ingoing and outgoing light ray geodesics if viewed by an observer far from the event horizon, where r approaches infinity. The incoming light rays seem to never actually enter the event horizon, and rather take an infinite amount of time to cross into the black hole, shown as the line approaches positive infinite time as it nears the radius at r = 2M[40,36]. The outgoing light rays cannot escape the black hole and will end up at the singularity, which is why the black hole appears dark, but light rays outside of the event horizon can still escape from the black hole's pull[40].

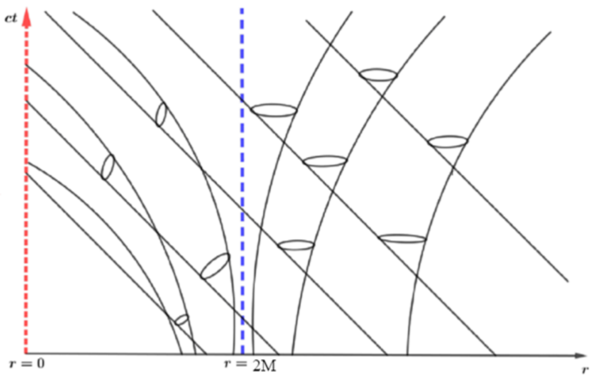

Figure 4.C Eddington-Finkelstein Diagram of a Schwarzschild Black Hole with Light Cones from[41]

Figure 4.B shows the ingoing and outgoing light ray geodesics if viewed by an observer who is nearing the event horizon at r = 2M and falling into the black hole. The incoming light rays do not appear to freeze in time and instead take a straight and direct path into the black hole, arriving at its singularity[36]. The outgoing light rays behave the same as they would for a distant observer: they are unable to escape unless they are outside of the event horizon[40].

Figure 4.C shows the light cones for ingoing and outgoing light rays, and how the black hole distorts their appearance. On the left side of the event horizon, the light cones start to rotate more the farther they get to the horizon until they are turned completely towards the singularity[36]. On the right side of the diagram, as the geodesics for outgoing light rays appear to straighten out, so do the light cones[40]. The farther to the right of the Schwarzschild radius an object goes, the straighter and less distorted the geodesic and light cone will be, as the curvature of spacetime will be far less and the spacetime transition into a flat spacetime[40].

The diagram's use of ingoing and outgoing coordinates makes sure that everything remains finite and consistent and can therefore create a clear and straightforward visualization of what happens near the event horizon. Even though it does not depict the entirety of spacetime, and other regions interconnected to black holes, the diagram specializes in understanding the event horizon, a key part of black holes[33].

Concerns and Limitations of Diagrams

As good as visual representations are at showing complex concepts, there is a limit to how much of the concept can be explained using diagrams as more research is needed to fully understand black holes and make sure the diagrams are not oversimplified and omit vital concepts. Since observations regarding regions inside the black holes have not been used to create visual representations, there is a high likelihood that they do not accurately represent black holes or their curvature of spacetime[35].

To maximally utilize black hole diagrams, they should be easily manipulated. However, the inefficiency to run situations on black hole diagram makes them difficult to use[42]. In addition, visual representations can often prove to be confusing rather than clarifying a concept in an understandable manner[43]. Since the diagrams do not offer a complete understanding of how objects interact with a black hole's curvature of spacetime, they can only be improved upon with discoveries that need to be first mathematically represented before being visually depicted. The more complexity there is for a concept, the more complicated the mathematical formulas to explain it will be, and the more detailed diagrams will become. As diagrams get better at representing more complex concepts, they may also lose their simplicity and stray away from being effective for new learners.

Future of Visual Representation and Research

Recent research in scientific education tools for physics include using online modules as a method of teaching[5]. Computer programs can be utilized to manipulate certain aspects of a diagram, such as writing the axes for a graph correctly, and ultimately removing the need for people to manually do it[42]. Some important features of diagrams that can be automated are the ability to effortlessly switch from the coordinates of one event to another perspective and measure distances on the diagram[42]. One example of an online tool is Physlets, which is an interactive educational tool where virtual experiments can be run, and the environment can be manipulated[44].

In addition to online diagrams, simulations can be utilized for a more interactive and efficient learning experience. In a study addressing students' difficulties with understanding physics, the proposed solution was to use interactive simulations as a new educational approach[1]. In a study exploring a new educational approach of teaching physics, the groups taught using simulations and interactive visualizations performed significantly better than those taught without[6]. With technology continuing to advance, computer simulations of three-dimensional models can become more viable than flat representations. For example, the OpenRelativity open-source toolkit by MIT Game Lab allows anyone to create three-dimensional games and simulations to help visualize complex physics topics[45]. The game "A Slower Speed of Light" teaches learners special relativity as the gameplay becomes harder, providing an engaging and interactive way to learn[45]. With three-dimensional space, learners would be able to understand the curvature of spacetime and physics of black holes with even more ease and accuracy to how they appear, instead a common two-dimensional depiction.

Analysis

As shown by Figure 5, the Eddington-Finkelstein, Kruskal-Szekeres, and Penrose Diagrams each have their unique strengths and weaknesses, and each should be considered based on the situation. The Eddington-Finkelstein has the most educational value for new and intermediate learners who want to learn about the basics of black hole physics, but the Kruskal-Szekeres and Penrose Diagrams both show the four spacetime regions and therefore can better illustrate a black hole's interaction of spacetime. The latter two diagrams, however, are more complicated and therefore less approachable for a new learner.

The Eddington-Finkelstein and Kruskal-Szekeres diagrams of a black hole both accurately show how black holes specifically affect the curvature of spacetime. The Eddington-Finkelstein diagram is still able to represent distant parts of spacetime, like the Penrose Diagram, as opposed to the Kruskal-Szekeres diagram. The Penrose diagram can show infinite distances compacted into a finite space, which neither of the other diagrams can do. Theoretically, the Eddington-Finkelstein diagram would be the most effective for new learners due to its simplicity and focus on the event horizon, but user studies need to be conducted to test this hypothesis.

Conclusion

Researchers need to communicate their findings and key ideas to others in the scientific community and avid learners in a way that makes it simple for everyone to understand. The best way to ensure interactive and effective teaching is through visual representations, but making sure to use the right ones is also important.

More research is needed to determine which diagram has the most educational value and is the most effective at teaching concepts about a black hole's curvature of space time. To assess the pedagogical effectiveness, a user study must be done that teaches 4 groups of people using traditional methods and each of the three diagrams, and later testing the knowledge of each group.

Eventually, visual representations can become the main form of communicating ideas, instead of mathematical formulas, and connect the more experienced researchers in the field to new yet avid learners in the scientific community who can and will learn, regardless of experience.

Acknowledgments

I would like to thank the Lumiere Program for providing me with the resources necessary to succeed and my mentor Marina David at KU Leuven for giving me guidance and supporting me throughout the paper.

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Diagram	Strengths	Weaknesses	Educational Value
Penrose Diagram	• Compacts the infinite region of spacetime into a finite representation • Shows how spacetime regions are connected to each other and through event horizons	• Does not accurately depict distances in spacetime	• Good for showing idea of infinity and behavior of objects due to curvature • Good for showing how regions are related • Advanced due to being more abstract
Kruskal-Szekeres Diagram	• Shows the four spacetime regions and removes coordinate singularity at event horizon • Shows structure of spacetime, including singularities and	• Warped scaling and distance that is unintuitive spatially	• Good for teaching the full spacetime structure, including region inside the event horizon • Require a mathematical background to fully utilize
Eddington-Finkelstein Diagram(s)	• Simply represents ingoing and outgoing light ray geodesics and light cones • Shows what different observers see based on their location	• Does not show the four spacetime regions	• Good for showing the concept of the event horizon and how it is one way • Intermediate/Beginner to introduce idea of black hole, cannot be used for all situations

Figure 5 Table of Diagrams and Strengths, Weaknesses, and Educational Value

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