Machi Koro II: A Continuous-Time Markovian Investigation

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Machi Koro II, otherwise known as 'Dice Town,' is a quick dice-rolling board game that allows players to build their own coin-generating city against others. Captivating players with its fun designs and need for technique, we sought to explore the optimal strategies for gaining landmarks and winning the game. Through a Markovian construction and examination of each establishment, or playing card, we uncovered patterns within each card color, the number of players, and the probability of rolling either one dice or two. Our findings found that blue establishments ensured a consistent accumulation of coins through the game for all amounts of players and within rolling one dice or two. On the other hand, red cards bested blue ones but only in early-game scenarios with a greater number of players. Green cards were also very effective in early-game situations but for a smaller number of players. We also analyzed the unique combo cards, revealing their potential for high return under varying conditions. For new players and old, Machi Koro II is a great tactical game, but statisticians may find themselves with a frequent upper hand.

Keywords: Markov Chain, CTMC, Machi Koro II, Board Game, Stochastic Process, Game Strategy, Statistics

Introduction

Building upon the success of its predecessor, Machi Koro II's fast-paced dice-rolling game combines strategic thinking and dynamic gameplay to create the highlight of modern board gaming. Translated to either 'Dice Town' or 'Around Town,' the game uses both tactics and 'luck' to pit players against each other to build their city the quickest[1]. With over a million copies sold, multiple game of the year nominations and wins, the game is a common staple amongst board game lovers of all ages[1]. Machi Koro II's allure may lie in its colorful artwork and accessible gameplay, but beneath the designs lie a rich landscape of probabilistic intricacies waiting to be explored.

Machi Koro II, a stochastic board game, similarly to the well-known Monopoly, centers on player purchases and fund savings to invest in city developments. Contrary to the original Machi Koro, each player begins with a small amount of money to which the players can either buy cards or save. The market cards, known as establishments which are available for purchase, vary between prices, rewards, corresponding colors, abilities, and activation. There are a total of 21 available establishments with 20 landmarks. After the initial phase of buying and saving coins at the start of the game, players take turns rolling either one dice or two dice, depending on what establishments they have purchased to earn coins. Half of the establishments are activated through numbers rolled on one dice, 1-7, while others are on two, 8-12. The aim of the game is to acquire enough coins to buy three landmarks which cost 12 coins each. The first to three landmarks wins.

Due to Machi Koro II's (MK2) stochastic nature, we must adopt a stochastic statistical method to analyze the benefits of each establishment to earn enough coins for landmarks. Markov Chains, a common stochastic process, predict the probability of an event out of a sequence based on the state of the previous event[2]. A key fact of Markov Chains is that future predictions of an event depend on its current situation, making them memoryless and thus useful in analyzing stochastic board games such as Machi Koro and Monopoly. A player's decision in these types of games is influenced by their current position, yet another benefit for MK2.

Markov Chains are characterized as either continuous-time, for constant transfers between events, or discrete-time, for incremental transfers between events. We will use continuous-time Markov's because in MK2, players make decisions constantly throughout the game; buying establishments, rolling dice, making payments, earning income, etc. Because these actions occur at any time, there is a continuous flow of state transitions. As such, Continuous-time Markov chains (CTMC) are closer in nature to MK2 than discrete Markov Chains and utilizable for this research.

In creating probability matrices of various amounts of players to the number of coins they can earn for each card, we can determine whether a certain subset of establishments is most beneficial for attaining landmarks. We consider questions such as "Does an establishment's upfront cost matter?", "Does it's color?", and more. These results will allow the player quicker acquisition of coins, better selection of establishments, and bragging rights to game night of this well-played yet under-analyzed board game. Ultimately, this paper concentrates on finding the optimal strategy in particular establishments for MK2 to win.

Literature Review

Markov Chains, named after their inventor, Andrey Markov, in 1906, have long been used in numerous real-world applications, including biology, physics, economics, and computer science. These chains, fundamental in probability theory, are effective mathematical tools for modeling dynamic situations contingent on the current state[2].

Mathematically, Markov Chains consist of a transition matrix and separate set of states. The transition matrix represents the probabilities of transitioning from one state to another while the set of states are the distinct situations or events one is predicting. For example, denote vector $S = S_1, S_2, \ldots, S_n$ as the set of possible states and transition matrix P as an *nn* where P_{ij} represents the probability of transitioning from state S_i to state S_j.[3]

$$P = \begin{bmatrix} P_{1\,1} & P_{1\,2} & \cdots & P_{1n} \\ P_{2\,1} & P_{2\,2} & \cdots & P_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & P_{n\,n} \end{bmatrix}$$

To find the new state distribution, *New State Distribution=Inital State Distribution* $\times$ P^t where t is the number of transitions. Figure 1 below represents an example Markov Chain with its transition matrix and a visual representation of the situation.

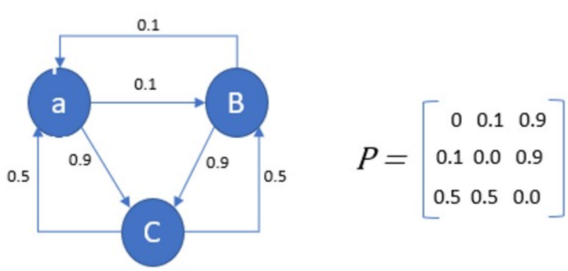

Fig. 1 Visual Representation of a Markov Chain[4]

As seen, the probability of one event to another is based solely on its current state and not on its previous places. This is the basis of the mathematics of a Markov Chain which are broadly categorized into two main types: discrete and continuous[2].

The two types of Markov Chains, Continuous-time (CTMC) and Discrete-time (DTMC), only differ in time intervals. Compared with CTMCs, DTMCs operate in discrete time steps where the system transitions from one state to another at fixed intervals[3]. CTMCs include transitions at any nonnegative real number. This means that the states are still fixed, but the time it takes to move from one state to another follows an exponential distribution pattern where transitions happen randomly[5]. CTMCs are a blend of DTMCs and the Poisson process, which is another method of handling random events over time. While the Poisson grows indefinitely, CTMCs do not and therefore have stable behaviors over time[5].

For MK2, CTMCs are a more applicable model despite the distinct turns in board games. This is because MK2's purchases, buys, and rewards are received and given continuously rather than strictly at the end for each turn. For example, players decide on purchases and receive rewards in a manner that reflects ongoing decision-making rather than discrete steps. Rewards are given immediately, which allows for purchases or other exchanges of coins to be made between turns, making DTMCs ineffective to model with. CTMCs more effectively capture this continuous decision-making of MK2 over the DTMCs, allowing for a more accurate modelling for our research.

There have been many previous studies on Markov Chains in other popular board games—notably Monopoly and Risk. By constructing Markov matrices, these investigations aimed to uncover insights on optimal strategies, resource management, and potential game outcomes. For Monopoly, researchers examined benefits of players on any given place by modeling the probability of ending a turn on a given monopoly space as a Markov chain. Doing so allows analysis on the amount of possible rent money made throughout the game per property over multiple rounds. Due to its stochastic nature, Markov chains were able to represent the many rounds of the game[6]. The same idea applied to Risk, another stochastic game, on whether a player should attack an army to acquire territories based on the number of armies a player has[4]. Based on previous research on other board games, stochastic ones, with representations of the many factors in gameplay, Markov chains have proven successful for finding possible unverified strategies and applicable in our research for MK2.

For our research, we hope to apply the lessons of previous Markovian research on board games with the widely used CTMCs to identify the benefits of each card in MK2 to master the game under different conditions.

Methodology

Our investigation in identifying optimal strategies for MK2 focuses on building Markov Chains for each establishment's turnout of coins for a certain number of rounds. To do so, we must construct the input vectors (initial state matrix) and transition state matrices for all establishments under certain criteria.

MK2 includes a variety of different establishments with varying effects and functions. There are four colors of establishments: blue, red, green, and purple, denoting specific abilities. For the blue establishments, if any player playing rolls the num-

Table 1 Input Vector and Transition Matrix for Vineyard (100% 1-Dice)

Card	Vineyard															
Color Type:	Blue															
Trigger:	2 coins on any 2														Cost+12	
2-Player (1-Die 100%)		0	1	2	3	4	5	6	7	8	9	10	11	12	13 (or higher)	
	Input Vector:	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
	Transition Matrix	0.69444444	0	0.27777777	0	0.02777777	0	0	0	0	0	0	0	0	0	1
		0	0.69444444	0	0.27777777	0	0.02777777	0	0	0	0	0	0	0	0	1
		0	0	0.69444444	0	0.27777777	0	0.02777777	0	0	0	0	0	0	0	1
		0	0	0	0.69444444	0	0.27777777	0	0.02777777	0	0	0	0	0	0	1
		0	0	0	0	0.69444444	0	0.27777777	0	0.02777777	0	0	0	0	0	1
		0	0	0	0	0	0.69444444	0	0.27777777	0	0.02777777	0	0	0	0	1
		0	0	0	0	0	0	0.69444444	0	0.27777777	0	0.02777777	0	0	0	1
		0	0	0	0	0	0	0	0.69444444	0	0.27777777	0	0.02777777	0	0	1
		0	0	0	0	0	0	0	0	0.69444444	0	0.27777777	0	0.02777777	0	1
		0	0	0	0	0	0	0	0	0	0.69444444	0	0.27777777	0	0.02777777778	1
		0	0	0	0	0	0	0	0	0	0	0.69444444	0	0.27777777	0.02777777778	1
		0	0	0	0	0	0	0	0	0	0	0	0.69444444	0	0.3055555556	1
		0	0	0	0	0	0	0	0	0	0	0	0	0.69444444	0.3055555556	1
		0	0	0	0	0	0	0	0	0	0	0	0	0	1	

Table 2 Input Vector and Transition Matrix for Vineyard (80% 1-Dice)

2-Player (1-Die 80%)		0	1	2	3	4	5	6	7	8	9	10	11	12	13 (or higher)	
	Input Vector:	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
	Transition Matrix	0.74151234	0	0.23919753	0	0.01929012	0	0	0	0	0	0	0	0	0	1
		0	0.74151234	0	0.23919753	0	0.01929012	0	0	0	0	0	0	0	0	1
		0	0	0.74151234	0	0.23919753	0	0.01929012	0	0	0	0	0	0	0	1
		0	0	0	0.74151234	0	0.23919753	0	0.01929012	0	0	0	0	0	0	1
		0	0	0	0	0.74151234	0	0.23919753	0	0.01929012	0	0	0	0	0	1
		0	0	0	0	0	0.74151234	0	0.23919753	0	0.01929012	0	0	0	0	1
		0	0	0	0	0	0	0.74151234	0	0.23919753	0	0.01929012	0	0	0	1
		0	0	0	0	0	0	0	0.74151234	0	0.23919753	0	0.01929012	0	0	1
		0	0	0	0	0	0	0	0	0.74151234	0	0.23919753	0	0.01929012	0	1
		0	0	0	0	0	0	0	0	0	0.74151234	0	0.23919753	0	0.01929012346	1
		0	0	0	0	0	0	0	0	0	0	0.74151234	0	0.23919753	0.01929012346	1
		0	0	0	0	0	0	0	0	0	0	0	0.74151234	0	0.2584876543	1
		0	0	0	0	0	0	0	0	0	0	0	0	0.74151234	0.2584876543	1
		0	0	0	0	0	0	0	0	0	0	0	0	0	1	
	Rotation 1:	0.74151234	0	0.23919753	0	0.01929012	0	0	0	0	0	0	0	0	0	
	Rotation 2:	0.54984055	0	0.35473584	0	0.08582318	0	0.00922829	0	0.00037210	0	0	0	0	0	
	Rotation 3:	0.40771356	0	0.39456151	0	0.15909738	0	0.03421449	0	0.00413884	0	0.00026702	0	0.00000717	0	
	Rotation 4:	0.30232464	0	0.39009630	0	0.22021565	0	0.07103730	0	0.01432203	0	0.00184800	0	0.00014903	0.000007006329	
	Rotation 5:	0.22417745	0	0.36157653	0	0.26243458	0	0.11287508	0	0.03185990	0	0.00616643	0	0.00082882	0.000081177673	
	Rotation 6:	0.16623034	0	0.32173615	0	0.28541110	0	0.15344683	0	0.05568635	0	0.01437067	0	0.00270415	0.000414369094	
	Rotation 7:	0.12326185	0	0.27833322	0	0.29180095	0	0.18825868	0	0.08350183	0	0.02693607	0	0.00651679	0.001390572715	

ber the card displays, the card-holder receive the rewarding number of coins. For red, the card-holder steals coins from the player who rolled the number; for green, it is the opposite with solely the card-holder needing to roll the denoted number; lastly, for purple, these establishments are primarily exchange cards with various effects on all other cards. For our purposes, we neglected all purple establishments because they are majorly not based on net coins but on exchanging cards.

Apart of the green establishments, there is a separate category of combo cards. In such, the card-holder earns more coins depending on how many of a certain subset of establishments the player has. For example, Furniture Factory is a combo card where the card-holder earns four coins for each establishment that contains a gear icon. In the full deck, variously colored establishments, including the Mine and Forest, contain the respective icon which is triggered when the card-holder rolls the number on the combo cards. As such, the combo cards function differently from the rest of the establishments and will be analyzed separately.

Additionally, each landmark costs minimum 12 coins so we consider all landmarks worth 12. They also have various effects themselves ranging in changes to net coins to player actions, but because we cannot predict which random landmark, each with very different effects on the cards that the card-holder may or may not have, we ignore all landmark effects similar to our reasoning for neglecting purples. Furthermore, the game is won by acquiring 3 landmarks, however, in our analysis we compared cards based on 1 landmark since the probability process will repeat similarly if done up until 3 landmarks.

Lastly, we created multiple Markov Chains for each establishment based various differing factors. The game is designed to be played with between 2-5 players, so we constructed matrices based on 2, 3, and 4 players. As well, during the game, the player has the choice of rolling two dice or one dice. To combat the uncertainty of a player randomly rolling one or two dice throughout their gameplay, we built five transition matrices for each card with a 100%, 80%, 50%, 20%, and 0% probability of rolling one dice versus two.

Ultimately, our research aimed to compute the probabilities of each establishment acquiring one landmark, comparing between colors, number of players, and probability of rolling with 1-dice.

To begin, we needed to create the input vector. Since the goal of the game is to reach 12 coins, the cost of a landmark, our input vector should represent the number of coins a player has at the start of each round, or transition. However, each card has different upfront costs which add onto the landmark's costs, so each matrix for every card has different dimensions. For all establishments, the input vector dimension is $1 \times n$ where $n = 12 + Cost$ of $Card + 1$. The plus 1 accounts for the 0. Because we assume that the player uses all coins at the start of the game and begins the rounds with 0 coins, the input vector begins at 0 as $[1, 0, 0, ..., 0 \times n]$.

Next, we needed to create the transition matrix, representing the probabilities of earning coins for a particular card. Similar to the construction of the input vector, the transition matrix's dimensions are $n \times n$ where each column represents the expected number of coins after one rotation round (your turn and everyone else's turn) and the row represents the current number of coins that you had starting the rotation. The matrix consists of probabilities that a player will end with a certain number of coins after a rotation.

Now, to obtain our new vector, or our new current standing of coins, we must multiply the input vector by the transition matrix. Doing this once represents one round, we repeated this seven times to obtain results after seven rounds, a reasonable number of rounds to earn enough coins for landmarks after our own experiences in playing the game.

At this point, we compared within each player scenario and condition of dice for each color and specific card. Below is an example of how we constructed the input vector and transition matrix for the blue establishment, Vineyard for 2 players at 1-dice 100%. Vineyard costs 1 coin at the start and is triggered by the roll of a two. As such, our input vector for Vineyard will be 1×14, beginning at 0, and our transition matrix is 14×14.

To construct the probabilities, we needed to determine the probability of obtaining a certain number of coins based on how many coins the card rewards. For example, the first slot of the transition matrix represents obtaining no coins for one dice on a card that is only triggered on one number, which is two, out of six on a dice. To earn no coins, one would need to roll any other number than two, making the probability of rolling a 2 equal to $\frac{5}{6}$. However, because this card is blue, either of the two players in this case needs to roll a 2, so the probability of either player rolling a 2 is $\left(\frac{5}{6}\right)^2 = 0.6944$.

If we wanted to find the probabilities for the same card and scenario but at an 80% chance of rolling 1-dice, we would follow a somewhat similar process. To earn no coins, instead of using $\frac{5}{6}$, we would use $\left(\frac{5}{6} \times 0.8\right)^2$ because there is an 80% chance of using 1 dice versus 2. The process repeats similarly for 50%, 20%, and 0% of rolling with 1 dice. Figure 1.3 depicts the 80% version of Vineyard in the same conditions as the 100% one—reflecting various results.

We analyzed the combos a little differently. Since they cannot be pooled in with the other cards, they need to have their own analysis because their matrices represent different factors than the other establishments. Their matrices are instead based on the possible number of triggers—the expected value of a random variable through the expected value formula. The triggers were calculated in the same way as the other cards, but instead represent the possible number of triggers where $\mu(x) = \sum x \times p(x)$ after a certain number of rotations. After a certain number of triggers, we compute the expected triggers of each combo card, we can calculate the expected gain from each combo card. By multiplying the expected probability of being triggered in a certain scenario by the gains per trigger, we can get the gross expected gain where the sum is $\mu(x) \times$ gains per trigger. We followed this procedure in every dice probability situation as well as based on the cardholder having either 1, 2, 3, or 4 cards that the holder's combo can trigger. So, the result of the combos is the actual coin addition over 5 rounds instead of a probability of acquiring enough coins to earn landmarks like the other establishments. Below are the total expected matrices for the combo cards with 1 card in combo.

We used this process of determining probabilities, altered for each card and number of obtainable coins, for the rest of the cards on all conditions of rolls and players using Excel and MATLAB on Google Colaboratory for modeling.

Results & Analysis

Using these methods, we were able to compute the end of 7th round probabilities for all blue, red, green, and combo cards after the 5th round as well as successfully compare them against each other. The following figures list all of the probabilities for each respective color per number of players and probability of using 1 dice.

These lists (Figures 1.5-1.8) are a record of our probabilities as an overall reference to each of the calculated probabilities of each card. Below (Figures 1.9-2.3) we graphed each possible card color versus its possible players and dice percentage. These visualizations reveal various results concerning card color and 1-dice percentage. By comparing all of the cards in 2-players at respective 1-dice percentages, we can see the benefits of certain cards over others at different points in the game (assuming players typically start the game rolling 1 dice and progress to 2).

Each of these graphs represent the establishment's probability of gaining a landmark for 2-players at various dice percentages

Table 3 End Probabilities of 1 Card in Combo Between Different Dice Percentages)

Est. Cost = (Other Card Cost * n + Combo Card Cost)			*NOTE: Food warehouse combo cards costs 1 or 2 each so assumed average of 1.5 per card.							
1-Card in Combo (100% 1-Die)										
Trigger Count	0	1	2	3	4	5	Expected Triggers	**Gain per Trigger**	Gross Expected Gain	**Est. Cost**
Flower Shop	0.401877572	0.401877572	0.160751028	0.032150205	0.003215020	0.000128600823	0.8333333333	3	2.5	3
Furniture Factory	0	0	0	0	0	0	0	4	0	8
Food Warehouse	0	0	0	0	0	0	0	2	0	3.5
Winery	0	0	0	0	0	0	0	3	0	4
1-Card in Combo (80% 1-Die)										
Trigger Count	0	1	2	3	4	5	Expected Triggers	**Gain per Trigger**	Gross Expected Gain	**Est. Cost**
Flower Shop	0.415453298	0.39894522	0.153237236	0.029429668	0.002826027	0.0001085494092	0.8055555556	3	2.416666667	3
Furniture Factory	0.868615786	0.124087969	0.007090741	0.000202592	0.000002894	0.000000016538171	0.1388888889	4	0.5555555556	8
Food Warehouse	0.868615786	0.124087969	0.007090741	0.000202592	0.000002894	0.000000016538171	0.1388888889	2	0.2777777778	3.5
Winery	0.893718635	0.101558935	0.004616315	0.000104916	0.000001192	0.000000005419228	0.1111111111	3	0.3333333333	4
1-Card in Combo (50% 1-Die)										
Trigger Count	0	1	2	3	4	5	Expected Triggers	**Gain per Trigger**	Gross Expected Gain	**Est. Cost**
Flower Shop	0.436502457	0.393567789	0.141942481	0.025596185	0.002307852	0.00008323403402	0.7638888889	3	2.291666667	3
Furniture Factory	0.697768775	0.260361483	0.038859922	0.002899994	0.000108208	0.00000161505582	0.3472222222	4	1.388888889	8
Food Warehouse	0.697768775	0.260361483	0.038859922	0.002899994	0.000108208	0.00000161505582	0.3472222222	2	0.6944444444	3.5
Winery	0.751418842	0.221005542	0.026000652	0.001529450	0.000044983	0.000000529221494	0.2777777778	3	0.8333333333	4
1-Card in Combo (20% 1-Die)										
Trigger Count	0	1	2	3	4	5	Expected Triggers	**Gain per Trigger**	Gross Expected Gain	**Est. Cost**
Flower Shop	0.458396273	0.386957893	0.130661106	0.022059667	0.001862179	0.00006287879558	0.7222222222	3	2.166666667	3
Furniture Factory	0.554928957	0.346830598	0.086707649	0.010838456	0.000677403	0.00001693508781	0.5555555556	4	2.222222222	8
Food Warehouse	0.554928957	0.346830598	0.086707649	0.010838456	0.000677403	0.00001693508781	0.5555555556	2	1.111111111	3.5
Winery	0.627851179	0.306268868	0.059759779	0.005830222	0.000284401	0.000005549289573	0.4444444444	3	1.333333333	4
1-Card in Combo (100% 2-Die)										
Trigger Count	0	1	2	3	4	5	Expected Triggers	**Gain per Trigger**	Gross Expected Gain	**Est. Cost**
Flower Shop	0.473473814	0.381833721	0.123172168	0.019866478	0.001602135	0.00005168178652	0.6944444444	3	2.083333333	3
Furniture Factory	0.473473814	0.381833721	0.123172168	0.019866478	0.001602135	0.00005168178652	0.3855430733	4	1.542172293	8
Food Warehouse	0.473473814	0.381833721	0.123172168	0.019866478	0.001602135	0.00005168178652	0.3855430733	2	0.7710861466	3.5
Winery	0.554928957	0.346830598	0.086707649	0.010838456	0.000677403	0.00001693508781	0.4060723256	3	1.218216977	4

in order for red, blue, and green after the 7th rotation. Do note that the green card probabilities will not change based on number of players throughout our analysis because they are only triggered by the card-holder. Additionally, each of the cards above have various upfront costs and returns, altering each establishment's effectiveness in earning enough coins for a landmark.

For 2 players, we see a general pattern (Figures 1.9-2.3) of blue cards achieving the highest probabilities versus the other cards for all dice percentages. However, green cards have an advantage over blue ones at 80-50% 1 dice situations. There is another pattern within the blue cards. The cards that are primarily triggered by one dice; Flower Garden, Vineyard, and Forest; have the highest probabilities compared to all other cards when 1-dice percentage is 100% and the contrary is true at 0% for Cornfield, Mine, and Apple Orchard. Specifically, Vineyard and Flower Garden are the most effective cards for 1-dice majority situations and the same for Corn field at 2-dice majority. This is due to each card's quick recovery from their upfront costs; Vineyard costs 1 coin, but returns 2; Flower Garden costs 2 coins but returns 2; and Corn field costs 2, but returns 3. Additionally, the probability of being triggered factors into each card's returns. Corn field is triggered by a 7, which has 6 combinations whereas, Apple Orchard, triggered by a 10, has only 3 combinations.

The reason behind blue card supremacy against other cards is because the main player gains coins based on other players as well, meaning that their returns are twice as frequent as the red player cards. This reasoning is why blue is more effective than red, which only gains coins based on the other player. The highest probability across all dice percentages for blue is ∼0.04 while the same for red is only ∼0.02.

Convenience stores' (G) equally competitve result with blue is a result of its specific costs and returns. It costs 1 but returns 3, a net gain of 2 which is higher than the net gain of most other cards (excluding Apple Orchard and Mine whose low probabilities are a result of their low landings from rolling frequencies). As such, Convenience store has an almost equal result, despite being

Table 4 Probabilities of Blue Cards with 2-4 Players

Card Name	1 Die Percentage	2 Players	3 Players	4 Players
Wheat Field	100%	0.000006063179586	0.006807478258	0.103923534
	80%	0.000000471611394	0.0009074943643	0.02295169919
*Probability of the card earning enough coins to buy a landmark	50%	0.0000000252563502	0.00001022219214	0.0005300572493
	20%	0	0.000000002211942491	0.0000002234434332
	0%	0	0	0
Vineyard	100%	0.004108722395	0.04791953404	0.1734705992
	80%	0.001390572715	0.01947763815	0.08364392999
	50%	0.0001516726653	0.002786583122	0.0155134827
	20%	0.000003958491204	0.00009490873737	0.0006854453479
	0%	0.00000003687373	0.000001052391028	0.000009031550402
Forest	100%	0.0006871967132	0.01556696104	0.08153124383
	80%	0.0004236704167	0.01036069443	0.05833043987
	50%	0.0001893206264	0.005188492897	0.04001578324
	20%	0.00007521599718	0.002307646078	0.01614159425
	0%	0.00003746645736	0.001238543548	0.009312415145
Corn Field	100%	0	0	0
	80%	0.00006399074682	0.0005350982077	0.002128789121
	50%	0.004234590195	0.02638664104	0.07888125215
	20%	0.0297274414	0.1382769795	0.3150398116
	0%	0.06897514678	0.2649513779	0.510521003
Mine	100%	0	0	0
	80%	0.00006755904242	0.004917972396	0.01111064256
	50%	0.002154485611	0.05492991945	0.1095269783
	20%	0.01150020474	0.1095269783	0.5442384882
	0%	0.02445656125	0.4165435308	0.7043807292
Apple Orchard	100%	0	0	0
	80%	0.000002270542767	0.00002093628995	0.00009175687295
	50%	0.0001831750269	0.001458529775	0.00552943986
	20%	0.001581914629	0.01087289233	0.03571398703
	0%	0.004234590195	0.02638664104	0.07888125215
Flower Garden	100%	0.004108722395	0.04791953404	0.1734705992
	80%	0.002207462294	0.02874103186	0.1152360022
	50%	0.0007312064436	0.01121697117	0.05250838524
	20%	0.0001813612344	0.003272793885	0.0179066899
	0%	0.00005649700191	0.001134989699	0.006891929346

Table 5 Probabilities of Green Cards with 2-4 Players

Card Name	1 Die Percentage	Any Players
Bakery	100%	0.0004572473708
	80%	0.0001465830308
*Probability of the card earning enough coins to buy a landmark	50%	0.00001703379195
	20%	0.0000003745770462
	0%	0.0000002790816472
Convenience Store	100%	0.002004029492
	80%	0.001221644531
	50%	0.0005145072937
	20%	0.001449874054
	0%	0.00007309148341

Table 6 Probabilities of Red Cards with 2-4 Players

Card Name	1 Die Percentage	2 Players	3 Players	4 Players
Sushi Bar	100%	0.002004029492	0.06897514678	0.2649513779
	80%	0.0006995226337	0.0297274414	0.1382769795
*Probability of the card earning enough coins to buy a landmark	50%	0.000000001345174867	0.004234590195	0.02638664104
	20%	0.000000816872428	0.00006399074682	0.0005350982077
	0%	0	0	0
Cafe	100%	0.000003572245085	0.004108722395	0.04791953404
	80%	0.0000007491540924	0.001761297247	0.02378609173
	50%	0.0000002790816472	0.0003520833635	0.005912365271
	20%	0.000000001633399673	0.0000361454878	0.0007524350823
	0%	0	0.000003958491204	0.00009490873737
Hamburger Stand	100%	0	0	0
	80%	0	0.00000003687373	0.004514292921
	50%	0.000000007788656582	0.00001726037837	0.0003789291033
	20%	0.000002090751581	0.0003520833635	0.005912365271
	0%	0.0000009969480425	0.001390572715	0.01947763815
Family Restaurant	100%	0	0	0
	80%	0.000000001345174867	0.000003624598678	0.000009651305627
	50%	0.0000008210295818	0.0001516726653	0.002786583122
	20%	0.000002203934502	0.002739697015	0.03438765671
	0%	0.00001050917865	0.009921627295	0.09622685606

green, than blue in the 1-dice majority situations.

As a result, players should rely on blue and green cards in 2-player situations. For the start of the game, when players

Table 7 Probabilities of Combo Cards with 1-4 Cards in Combo

Card Name	1 Die Percentage	1 Card in Combo	2 Cards in Combo	3 Cards in Combo	4 Cards in Combo
Flower Shop	100%	2.5	5	7.5	10
	80%	2.416666667	4.833333333	7.25	9.666666667
	50%	2.291666667	4.583333333	6.875	9.166666667
*Gross Net Gain of Coins	20%	2.166666667	4.333333333	6.5	8.666666667
	0%	2.083333333	4.166666667	6.25	8.333333333
Winery	100%	0	0	0	0
	80%	0.3333333333	0.6666666667	1	1.333333333
	50%	0.8333333333	1.666666667	2.5	3.333333333
	20%	1.333333333	2.666666667	4	5.333333333
	0%	1.218216977	2.436433953	3.65465093	4.872867907
Furniture Factory	100%	0	0	0	0
	80%	0.5555555556	1.111111111	1.666666667	2.222222222
	50%	1.388888889	2.777777778	4.166666667	5.555555556
	20%	2.222222222	4.444444444	6.666666667	8.888888889
	0%	1.542172293	3.084344587	4.62651688	6.168689173
Food Warehouse	100%	0	0	0	0
	80%	0.2777777778	0.5555555556	0.8333333333	1.111111111
	50%	0.6944444444	1.388888889	2.083333333	2.777777778
	20%	1.111111111	2.222222222	3.333333333	4.444444444
	0%	0.7710861466	1.542172293	2.31325844	3.084344587

are more likely to roll one dice, players should buy Vineyard, Flower Garden, and Convenience Store; for later in the game, they should rely on Corn Field. This is due to each cards' low cost, high gross reward (in comparision to their cost), and quick return (based on 2 players). However, this result may vary for more players. Below (Figures 2.4-2.8) are the same situations as above but for 3 players, allowing us to analyze 3 players the same as 2.

As compared with 2 players, we see some new patterns in Figures 2.4-2.8. The blues continue to maintain generally higher probabilities compared to other cards, however, in the 2-dice majorities, we see that Mine specifically has dramatic results. With 3 players, blue cards are triggered more often because there are more players. As such, it becomes more frequent to roll the more unlikely numbers, such as 11 and 12, which Mine is triggered on. This, coupled with a 2-coin net profit has led to the Mine's new advantage. For green, Convenience store has lost its competition with blue cards but Convenience store continues to be somewhat effective in the 80

Reds, mainly Sushi Bar (cost 2, take 3 from opponent) have amassed a dramatically high result from its previous performance with 2 players, the highest increase in probabilities. Triggered by 1, it is most powerful in the 1-dice majority, but does perform higher than Vineyard and Flower Garden for a few reasons. With more players, similar to blue, there are an increased number of chances to roll a 1 within a turn, increasing how often it is triggered. Additionally, reds require the card-holder's opponents to pay them instead of taking from the bank. As such, all opponents decrease by 3 at the same time that the cardholder increases by 4 (2 players give 2). However, since it is impossible to roll a 1 on 2-dice, we see very low performance in the 2-dice majority situations compared to blues.

As such, players in a 3-player scenario should continue to rely on blues, less on greens, and more on reds. In particular, Sushi Bar is very effective in the beginning of the game while Mine assumes that role later. Specifically, players should utilize Sushi Bar for its low cost and effective return at the beginning

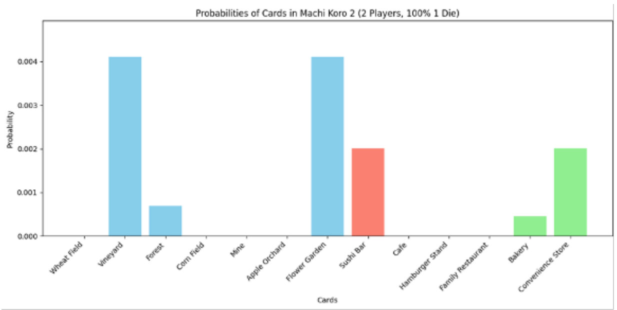

Fig 2: Probabilities of All Cards, 2 Players, 100% 1 Dice

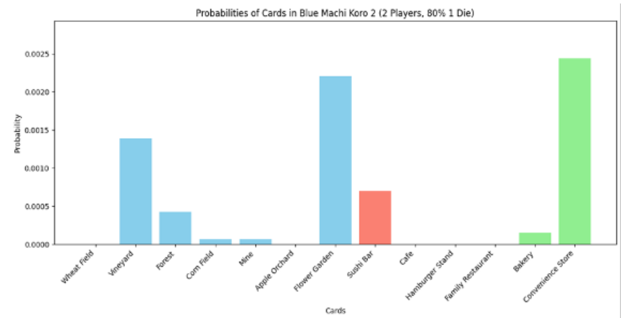

Fig 3: Probabilities of All Cards, 2 Players, 80% 1 Dice

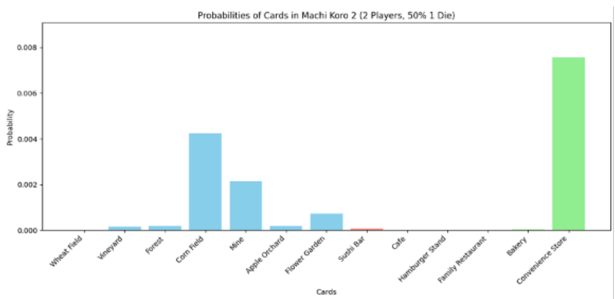

Fig 4: Probabilities of All Cards, 2 Players, 50% 1 Dice

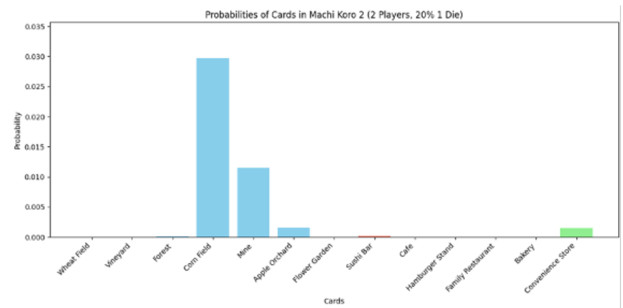

Fig 5: Probabilities of All Cards, 2 Players, 20% 1 Dice

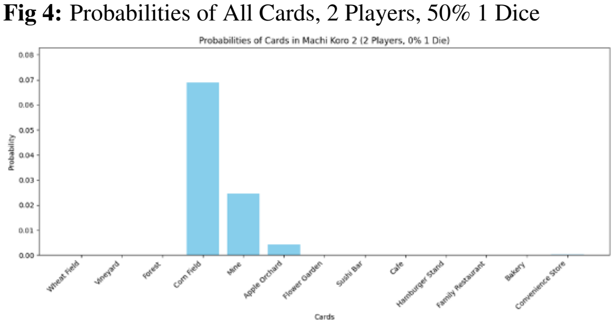

Fig 6: Probabilities of All Cards, 2 Players, 0% 1 Dice

of the game and assume Mine later on when rolling with 2 dice. However, the player can rely on a mixture of blue cards as a stable return of net gain throughout the game as blues seen in the figures continue to have a high average probability compared to reds and greens. Lastly, we have the 4-player scenarios (Figures 2.9-3.3).

Contrary to 2 and 3 player games, 4 players find more representative and competing cards, with at least a small benefit for all cards. However, 4 players follow a similar pattern to the 3 player games. As continued from the previous players, blue establishments are a stable and very effective choice for earning coins compared to reds and greens, primarily in the 2-dice majority situations as seen in 3 players.

The red cards, primarily Sushi Bar, gain a greater advantage in the 1-dice situation than the 3-player game due to the increase in players who can trigger Sushi Bar and more total coins gained with more players paying. However, we see that green cards have completely fallen away are no longer effective set of cards because they continued to have the same return as in 2 players while blues and reds increased in a nonlinear fashion. Players in a 4-person game should adopt a similar strategy as 3 player ones to gain landmarks quickest, relying on Sushi Bar (1 dice majority) and Cornfield/Mine (2 dice majority). Below is the analysis for the combo cards which depicts the expected gain for each combo card based on the cards in combo (Figures 3.4-3.7).

Here we analyzed each combo card against each other based on gross expected gain from between 1-4 cards in combo in possession. As expected, the greater number of cards in combo, the greater expected gain of combo cards. Besides for a slight overpower at 20% chance of rolling 1-dice, players, if they are going to use combo cards, should use Flower Shop throughout the game compared to other combo cards. Next, use Furniture

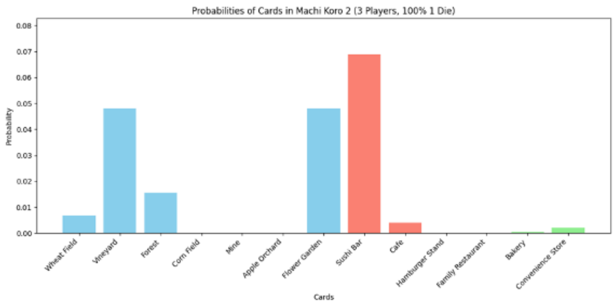

Fig 7: Probabilities of All Cards, 3 Players, 100% 1 Dice

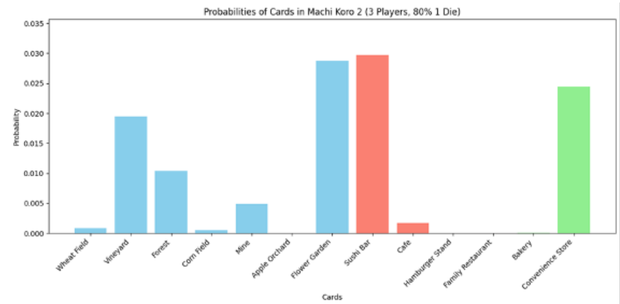

Fig 8: Probabilities of All Cards, 3 Players, 80% 1 Dice

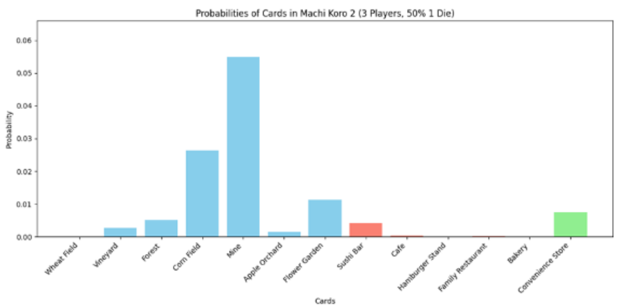

Fig 9: Probabilities of All Cards, 3 Players, 50% 1 Dice

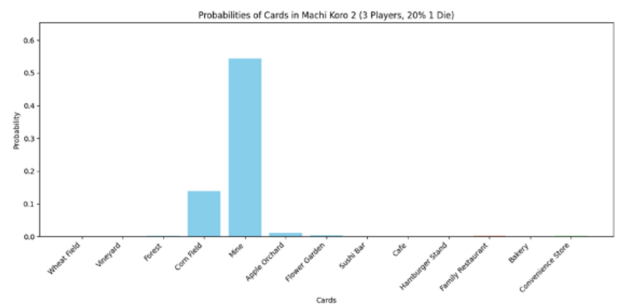

Fig 10: Probabilities of All Cards, 3 Players, 20% 1 Dice

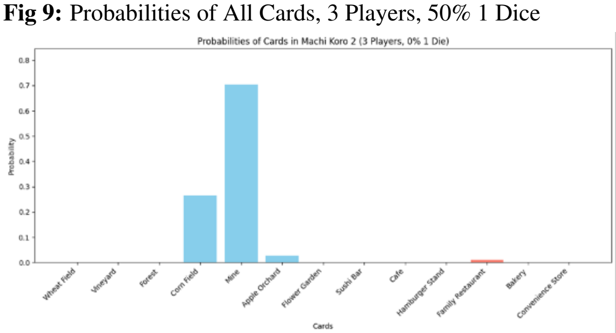

Fig 11: Probabilities of All Cards, 3 Players, 0% 1 Dice

Store, Winery, and finally, Food Warehouse. Combo cards can be very effective but players must analyze how many cards in combo they have before acquiring a combo card because they can be costly—requiring up to 4 coins for Furniture Factory.

Next, we compared each individual card against all numbers of players and dice situation to determine when individual cards are most effective (Figures 3.8-4.2). Putting all the cards together at each player number allows us to view which cards out of them all are most effective at different numbers of players. In doing so, players can adjust the cards they choose based on how many players are present.

For 100% and 80% 1 dice scenario, we see the highest performances in Sushi Bar and Vineyard for any number of players. However, for 50%, 20%, and 0%, we see that Mine and Corn Field are the most effective compared to the rest of the cards. Ultimately, players should aim to acquire Sushi Bar and Vineyard at the start of the game (under the assumption that players roll with generally with one dice in the beginning of the game) and Mine and Corn Field later on while analyzing their cards in combo if they want to buy a particular combo card.

Discussion

In our pursuit of the strongest cards in MK2 we found many valuable insights into the dynamics of the game. However, like any study, there are limitations to consider. First, we neglected the purple cards as well as the impacts of landmarks. In real gameplay, the purple cards can swap between others and disrupt the probability models. The same applies for landmarks. Yet, these cards are but a small effect on the majority of the gameplay, allowing future players a strong chance to beat others.

Another limitation lies in the decisions of players. Humans do

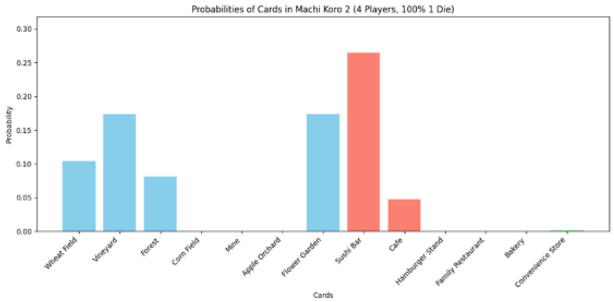

Fig 12: Probabilities of All Cards, 4 Players, 100% 1 Dice

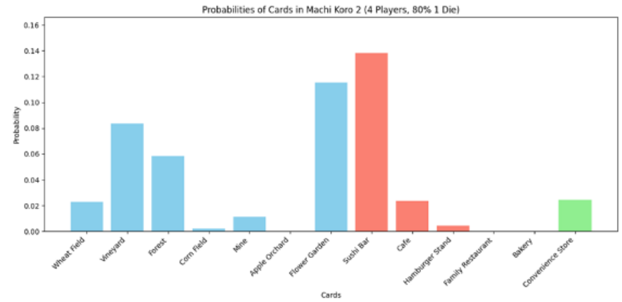

Fig 13: Probabilities of All Cards, 4 Players, 80% 1 Dice

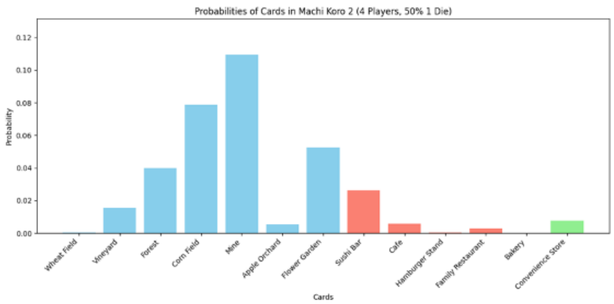

Fig 14: Probabilities of All Cards, 4 Players, 50% 1 Dice

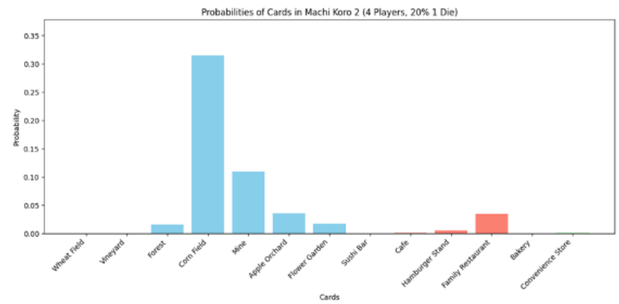

Fig 15: Probabilities of All Cards, 4 Players, 20% 1 Dice

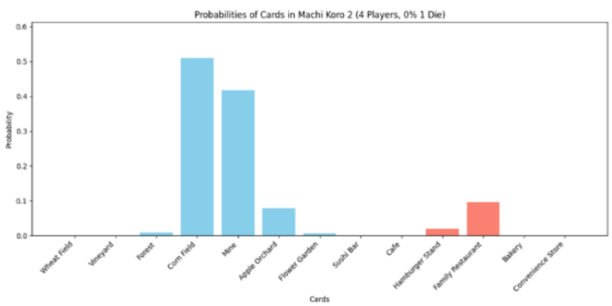

Fig 16: Probabilities of All Cards, 4 Players, 0% 1 Dice

not follow uniform playing habits and may adapt their strategies based on their current standings in the games. In reality, player strategies and in-game decision may vary based on individual preferences, experience levels, and game dynamics. While this research assumes that each player prioritizes maximizing coin accumulation for landmark purchases, in reality, others may try other techniques such as disrupting opponents' plans or more.

This study analyzed the possible strategies for Machi Koro II over the first Machi Koro because of its several improvements that enable a richer subject for analysis as well as its more popular and newer edition. Machi Koro II is a more complicated gameplay with more specific effects as well as a wider array of cards and rules that has not previously been researched upon. By tackling the harder of the two editions, we can bring wanted attention to the growing game.

Future work off of this research may be a dive into the effects of the purple establishments and landmarks in certain scenarios. Or, a study of a set of players playing similarly to these rolling and playing conditions to see how effective our results are in real gameplay. Regardless, our results offer a new chance to master in MK2 that has been previously untapped in.

Conclusion

With an increase in board game popularity, more families and friends are testing their hand in both nostalgic and modern board games. Machi Koro II is similar to Monopoly but has many of its own unique qualities that make it exciting during every game. However, there have been few statistical studies into these new games. As such, we examined the probabilities of various card outcomes under different game conditions, identifying key trends and highlighting recommendations for a swift win.

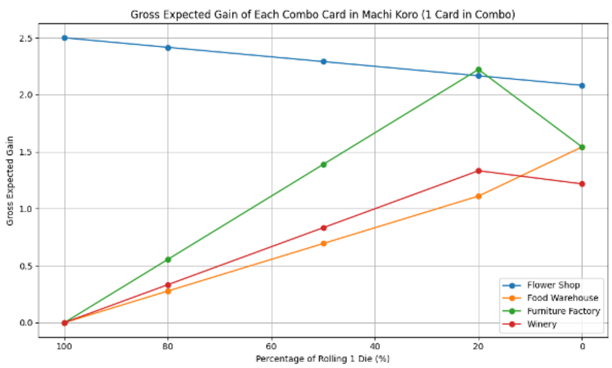

Fig 17: Gross Expected Game of 1 Card in Combo

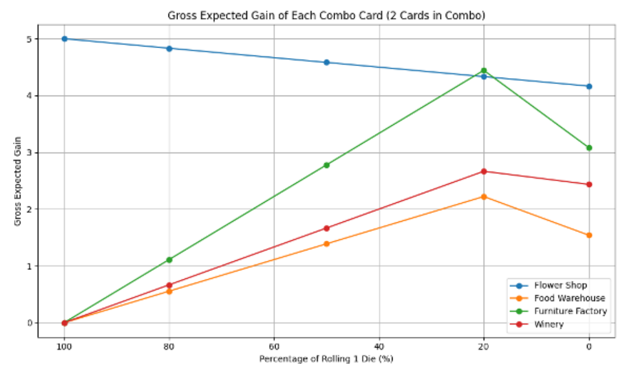

Fig 18: Gross Expected Game of 2 Cards in Combo

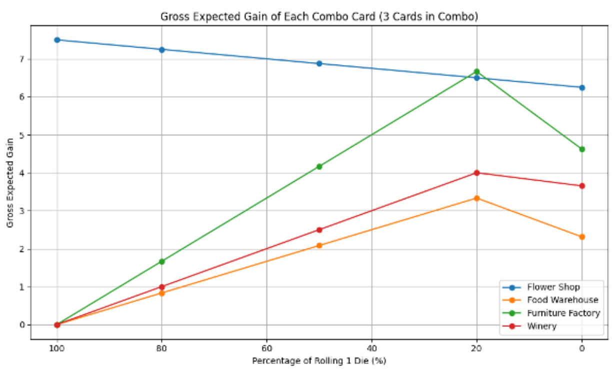

Fig 19: Gross Expected Game of 3 Cards in Combo

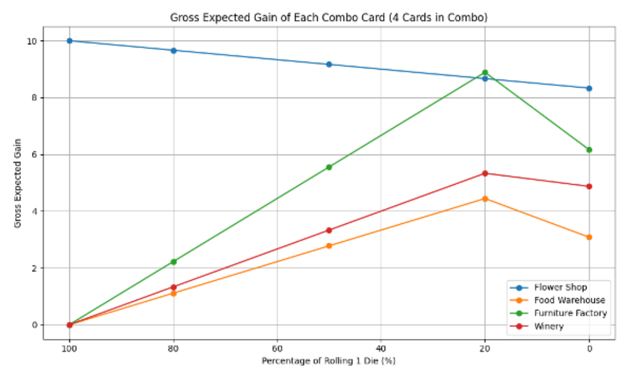

Fig 20: Gross Expected Game of 4 Cards in Combo

We saw that blue establishments tend to offer the most consistent coin generation for both the beginning and end of the game for all players and rolling 1-dice probabilities. However, we found that reds have proven to be more effective than blues with more players and earlier in the game. Conversely, green cards should be utilized with a smaller amount of people. Certain cards, such as Flower Garden (B), Vineyard (B), Corn Field (B), Mine (B), Sushi Bar (R), and Convenience Store (G) have proven to show steady returns on investment.

We note that the combo cards are up to the player's decision. If they have more than 1 card activated by a combo, they should buy the combo. However, the player needs to decide if they have enough in their total set of cards before buying. If they do consider buying, Flower Shop has the greatest coin profits.

Due to the limitations we experienced with this research, the conclusions we have come to are fair chances of improvement rather than adamant assertions that following a particular strategy will always result victoriously.

Future players should test our results to test their own luck with this guidance. Testers should gather between 2-4 possible players who have not researched strategies of MK2 and follow the respective advice per player and per the time of the game. Testers should utilize certain cards for the beginning of the game based on our results and the same for the end of the game. They should then record their results based on the other players and repeat without discussing any of the proposed strategies. This should also be repeated by other testers with different sets and numbers of people. Testers can also try to do the opposite of the advice in this research and record their respective results.

Our research has designed a framework and a guide for future MK2 players to loosely follow to earn the most landmarks and better their chances in winning. The purple cards, landmark effects, and in-game decision making may impact the results of the study, however, the main trends we identified will demonstrate themselves over the seven rounds and beyond. The next time someone proposes to play Machi Koro II on a game night, be sure to expect to be victorious by simply following the statistics.

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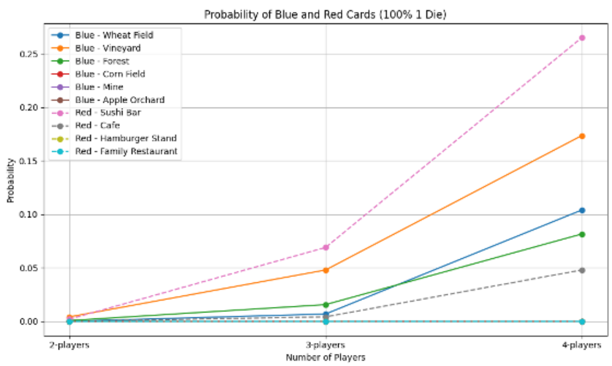

Fig 21: Probabilities of All Cards, All Players, 100% 1 Dice

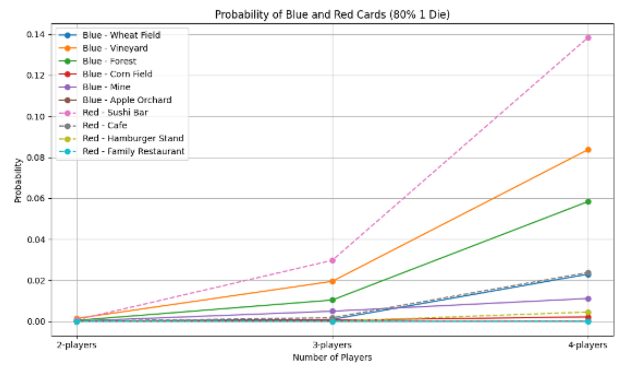

Fig 22: Probabilities of All Cards, All Players, 80% 1 Dice

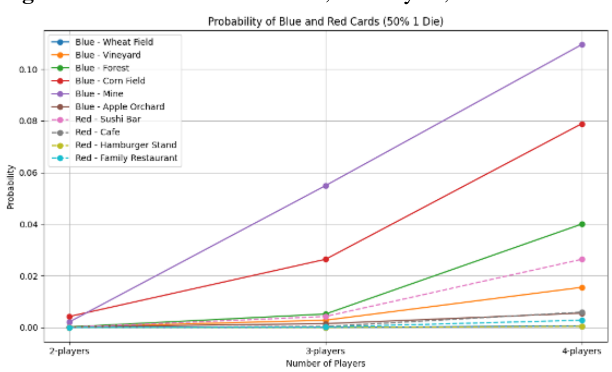

Fig 23: Probabilities of All Cards, All Players, 50% 1 Dice

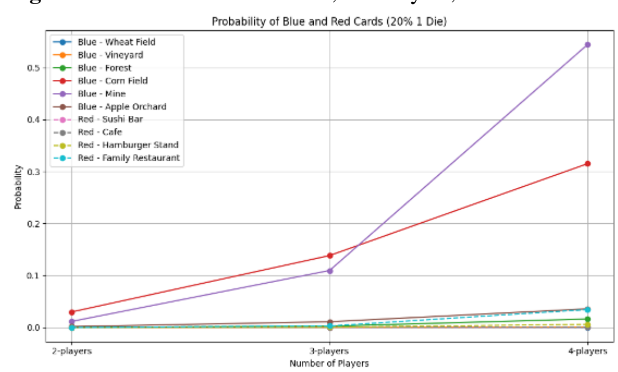

Fig 24: Probabilities of All Cards, All Players, 20% 1 Dice

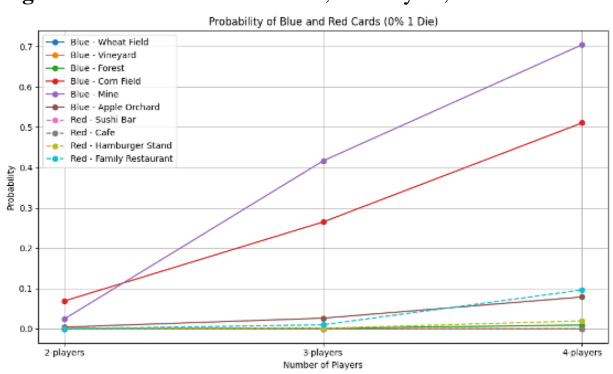

Fig 25: Probabilities of All Cards, All Players, 0% 1 Dice

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