Society of the 8-15th Century Middle East from the Lens of Mathematics

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This paper investigates the relationship between developments in Middle Eastern mathematics and the broader social conditions from the 8th to 15th centuries, demonstrating how mathematics functioned as a barometer of Middle Eastern civilization's rise and fall. During the Abbasid Caliphate's ascendance in the 8th and 9th centuries, Middle Eastern mathematicians focused on assimilating ancient Greek and Indian mathematical knowledge, illustrating the region's formative stage. The Golden Age from the 10th to 11th centuries witnessed a flowering of mathematical creativity and discovery fueled by increased resources, institutional support, and an intellectual culture that prioritized learning, reflecting Islam's zenith. However, after the Golden Age, political turmoil during the Mongol invasions and Black Death pandemic from the 12th to 14th centuries led to stagnation in mathematical progress while Europe and India advanced rapidly, exemplifying Middle Eastern decline. Nonetheless, mathematical astronomy advanced under Mongol rulers like Ulugh Beg in the 15th century, though Europe's Renaissance onset presaged a passing of the torch of intellectual leadership. Overall, developments in Middle Eastern mathematics closely tracked the rise and fall of Islamic civilization: periods of stability and prosperity bred mathematical innovation, whereas instability and decline stifled creativity, revealing mathematics as a sensitive gauge of the broader social conditions that either nourished or strangled intellectual life.

Introduction

This paper investigates the relationship between developments in Middle Eastern mathematics and the broader social conditions from the 8th to 15th centuries, demonstrating how mathematics functioned as a barometer of Middle Eastern civilization's rise and fall. During the Abbasid Caliphate's ascendance in the 8th and 9th centuries, Middle Eastern mathematicians focused on assimilating ancient Greek and Indian mathematical knowledge, illustrating the region's formative stage. The Golden Age from the 10th to 11th centuries witnessed a flowering of mathematical creativity and discovery fueled by increased resources, institutional support, and an intellectual culture that prioritized learning, reflecting Islam's zenith. However, after the Golden Age, political turmoil during the Mongol invasions and Black Death pandemic from the 12th to 14th centuries led to stagnation in mathematical progress while Europe and India advanced rapidly, exemplifying Middle Eastern decline. Nonetheless, mathematical astronomy advanced under Mongol rulers like Ulugh Beg in the 15th century, though Europe's Renaissance onset presaged a passing of the torch of intellectual leadership. Overall, developments in Middle Eastern mathematics closely tracked the rise and fall of Islamic civilization: periods of stability and prosperity bred mathematical innovation, whereas instability and decline stifled creativity, revealing mathematics as a sensitive gauge of the broader social conditions that either nourished or strangled intellectual life.

8-9th Century: Building a Solid Foundation

During the 8th-9th century, the Middle East had begun its rise as a great civilization under the Abbasid caliphate. They inherited a large and resourceful empire from the previous Umayyad rulers. As opposed to conquering as their predecessors did, the Abbasid caliphate focused on flourishing trade, arts, and sciences. The beginning of the Abbasids, because of this great environment for learning and advancement, allowed the Middle East under the Abbasids to cultivate great intellectual and cultural developments as well as world-renowned scholars and mathematicians, such as Al Khwarizmi the father of algebra, and Al-Kindi the father of Arab poetry. During this period, the Middle East was very prosperous, forming relationships with powerful nations with different cultures and achievements (Bentley, 2020)[1]. This change helped mold the Middle East into a region rich with intellectual advancement and was evident in many fields, among them mathematics. One prominent example of the sharing of knowledge between different societies was the Greco-Arabic translation movement where major mathematical Greek works (the birthplace of geometry) were translated into Arabic (Gutas, 1998)[2]. This translation effort aligned with the societal values and needs of the Abbasid Caliphate as selectively adapting Greek texts allowed useful ideas to spread while upholding Islamic beliefs, as the Abbasids incorporated knowledge from ancient sources to advance their empire in a culturally appropriate way. The translations supported their goal

of establishing a prosperous and intellectual center through the spread of ideas that could benefit their society. Furthermore, this allowed the Middle East to learn from the Greeks and pass down that knowledge to the next generation through schools. Furthermore, the Middle East was able to use what they learned from the Greeks to discover even more concepts. This development reflected the Middle East's ability to learn from other societies which demonstrates their highly adaptive and advanced approach. Besides mathematics, the Middle East employed this tactic of translation in other fields, which enabled them to learn the processes of the printing press from Chinese prisoners, enabling the copying of texts at an unseen speed (Bentley, 2020)[1]. The printing press allowed for even greater dissemination of knowledge, before the printing press all texts were copied by hand, meaning each copy was valuable and took a long time to create. This meant that these texts were distributed only to the wealthy or nobility, in fact before the renaissance in Europe, it was exceptionally rare to find a text written in vernacular or for entertainment (Bentley, 2020)[1]. While translating the roman philosopher Nicomachus of Gerasa's "Arithmetic", Thābit ibn Qurrah of northern Mesopotamia discovered a rule for finding Amicable numbers, two numbers each of which are the sum of the factors of the other number (Weisstein, 2022)[3]. Following the discovery of this trick, more mathematicians began researching such numbers. This breakthrough reflected the society at that time as it showed that although the empire was advanced for its time, there was still so much unknown, and countless ideas lay buried or hidden in ancient texts, just waiting to be discovered. One pivotal innovation of this time was the Hindu Arabic numerals, which are used to this day (Britannica, 2017)[4]. This discovery marks the end of the Middle East's rise and the beginning of its Golden Age, as the Hindu Arabic numerals would enable countless mathematical discoveries and would make mathematics more accessible to the general public due to its simplification. Besides numerals, 9th century Middle Eastern mathematicians commonly borrowed from Indian works due to their mathematical prowess which was unrivaled in the 9th century. Muḥammad ibn Mūsā al-Khwārizmī of the house of wisdom was one such mathematician, he wrote many books such as the "Book of Addition and Subtraction According to the Hindu Calculation" where he explained Hindu arithmetic (Britannica, 2022)[5]. However, Al-Khwarizmi's claim to fame was his book titled "The Compendious Book on Calculation by Completion and Balancing" also known as "Al-Jabr" where he was the first to establish Algebra as an independent discipline, this book provided an abundance of solutions to solving roots and polynomials (Du Sautory, 2008)[6], it was so well written and ahead of its time that until the 16th centuries, most European universities used it as the principle textbook. These two books had a monumental impact on Middle Eastern society and led to a tradition of arithmetic books which eventually led to the invention of decimals and fractions. Al-Jabr's creation reflected society as it alluded to a trend of passing down information to the next generation which would foster a better-educated youth. This is one of the many reasons why the Middle East would become exceedingly advanced, and why it would eventually take the lead in the study of mathematics. Apart from writing manuscripts, Al Khwarizmi was also a highly accomplished astronomer, who calculated how long the new moon would be visible, which signaled the beginning of the Muslim month. Al Khwarizmi used his vast knowledge of mathematics to calculate the movement of the sun, moon, and five planets known at the time (Britannica, 2022)[7]. This illustrates that in the 9th to 10th centuries, Mathematics was not seen as that important, and as a result in this era was still used primarily as a practical tool in other fields such as architecture where geometry was needed for exact measurements, or astronomy where algebra and trigonometry were incredible tools for seeing the unseen (Du Sautory, 2008)[6]. The 8th-9th century societies' attitude toward education was very positive, urging many to study as scholars and rise in social status or become merchants rather than remain as peasants, this helped foster a strong learning environment that raised many prominent scholars and intellectuals, this upwards social mobility encouraged perseverance in studies with the goal of climbing hierarchies. This motivated mathematical progress and application of theory to real-world problems over generations. Although the Middle East had made considerable progress and was making a name for itself in the world of mathematics, they were still far behind civilizations such as India where mathematics and learning were a key element of their culture, Indias legal systems, trade, and bureaucracy were all ahead of its time and were far more advanced than the Middle Easts. This was reflected by their Mathematical progress which overshadowed the Middle Easts in the 8-9th century. In the 8th century, the Indian mathematician Acharya Virasena was already giving explicit rules for the Fibonacci sequence, gave the derivation of the volume of the frustum using an infinite procedure, and understood logarithms and all of its laws. Similarly, Shridhara another mathematician found a rule for finding the volume of a circle and was able to create a formula for solving different quadratic equations. These principles were highly advanced and demonstrate just how far ahead Indian mathematics was compared to its Middle Eastern counterparts. Outside of mathematics, India also flourished in the sciences, and in the 9th century, a mathematician named Govindasvāmi discovered the Newton-Gauss interpolation formula centuries before the Europeans, a testament to how ahead of their time the Indian math and sciences were.

10-11th Century: The Golden Age Under the Abbasid Empire

The Abbasid Revolution in 750 CE marked the overthrow of the Umayyad caliphate and the move of the Islamic capital from Damascus to Baghdad. The new Abbasid caliphs were more open-minded and sought to centralize their vast empire. They embraced cultural exchanges and established an atmosphere of free inquiry. The Abbasid Revolution created a new intellectual culture as the caliphs embraced cultural exchanges and established institutions like the House of Wisdom under al-Ma'mun, centralizing scholarship and providing patronage that fueled collaborative advancement by mathematicians building upon classical Greek foundations and sparking the Golden Age of mathematical innovation. Major mathematical developments in fields such as algebra and trigonometry flourished due to the transition to an atmosphere of free scientific inquiry and access to foundations of the Greek mathematical tradition. During the Golden Age of the Middle East, the Abbasids were in the zenith of their time, arguably the most powerful empire on earth. The Middle East had become a prominent trading hub, with routes that spread to sub-Saharan Africa, Europe, and even the spice-rich southeast Asia, trade made the Abbasids rich. Unlike most empires such as China which looked down on Merchants and saw them as greedy and dishonorable, in the Middle East, merchants and traders were an honorable job and were pursued by many of the young population, this stems from the fact that the unifier and founder of the Islamic world Muhammad who to this day is one of the most respected figures in history, was originally a merchant who traveled as far north as Syria. This immense wealth allowed the government to build many schools and fund better education for the public (Bentley, 2020)[1]. Because of all the resources and schools, the Middle East bloomed into a great learning environment that raised many scholars, many of whom would have a groundbreaking impact on the world, this emphasis on education helped propel the Middle East into its golden age and continued to flourish due to the substantial funding. The government believed that mathematics was such an important school of thought, that they began investing large amounts of money and resources in commissioning the creation of many mathematical books and libraries to educate their empire better (Du Sautory, 2008)[6]. These actions reflected the society of the Middle East as it demonstrates the government's dedication to improving the country, investing money in learning and research rather than for conquest which could potentially damage the nation like so many other nations have done. These actions exhibit an advanced nation that prioritizes education, peace, and prosperity which greatly differed from the Ummayad empire before them, which had a reputation for conquest, and was mainly responsible for uniting the Middle East. A testament to the importance of this era is our math textbooks, which are mainly composed of concepts discovered or improved upon during the 10-11th centuries. Mathematics reflected this prosperity as scholars were no longer limited to researching one principle of mathematics, public colleges and libraries provided many enough resources to study multiple studies such as the three main subjects geometry, algebra, and arithmetic (Fraiser, 2020)[8]. As society began to become more complex due to the varieties of occupations and cultures, mathematics also became more complex, with algebra, number theory, and decimals emerging. Furthermore, many mathematicians looked past the 3 major studies and math's practical purposes, researching intangible and previously unknown concepts such as high degree polynomials, imaginary numbers, and calculus (Du Sautory, 2008)[6] this mirrored the societal growth as the Abbasids ushered in an era of unprecedented and unimaginable growth and infrastructure. This ability to study multiple principles of math alluded to the abundance of available resources in the region that gave civilians more social freedom/mobility than surrounding regions. As the Middle East now had a strong understanding of mathematics, many mathematicians began using their strong understanding of algebra to investigate Hindi and Greek equations or problems such as the Diophantine equation where the Middle East's advanced understanding of polynomials provided them with great support. This indicates a change in the Middle East's mathematical prowess as in a matter of centuries they have gone from learning from foreign countries to solving their questions. Outside of mathematics the Middle East was visibly becoming the most advanced nation as power in the Balkans and Mediterranean diminished. This Golden Age was so ahead of its time that many theorems that European mathematicians in the late renaissance and enlightenment would discover were already discovered or already being used by the tenth century Middle East. The Middle East was ahead of Europe mainly because the Islamic world embraced knowledge and intellectual pursuits as a virtue of their faith, which led to the founding of universities, libraries and houses of wisdom where scholars could study and make discoveries. Meanwhile, Europe was experiencing a lack of education and stability following the collapse of the Western Roman Empire. Additionally, the Middle East had access to ancient Greek and Indian scientific works that had been lost in Europe, allowing scholars to build upon past knowledge and make new advances. Examples of such theorems would be Fermat's last theorem (there aren't any rational solutions to $x^3 + y^3 = z^3$), whose special case was already beginning to be solved by Middle Eastern mathematicians in the first half of the 10th century. The Wilsons theorem which was discovered in 1770 (Hosch, 2012)[9], was used by Ibn al-Haytham born in the second half of the 10th century to solve problems about congruence. Pascal's triangle, invented in 1623 France had previously been proved in 1000 AD by Al-Karaji in his book containing proofs along with the binomial theorem and sum of integral cubes (Sesiano, 2021)[10]. These actions mirror Middle East's highly out-of-world technology and appearance in fields such as

architecture where their perfectly symmetrical domes would be an architectural masterpiece to be marveled at and would only be replicated in Byzantium and Europe during the Renaissance. Discoveries aside, existing mathematic concepts underwent significant development throughout the Abbasid Golden Age, mathematicians such as Abū Sahl al-Kūhī and Ibn al-Haythamhad leveled up from studying general Euclidean geometry to studying more complex geometric principles such as deriving the volume and area of conic sections, using their knowledge of conic sections, many of these mathematicians went on to investigate the optical properties of mirrors (Fraiser, 2020)[8]. This development showed that the Middle East was able to build on what it had and improve, parallels can be drawn to the Middle East's system of trade with foreign Kingdoms, which started as a few merchants and Sufi missionaries traveling to far away civilizations over time molded into a very well structured trade conducted along well-patrolled paths or sea routes which efficiently imported and exported goods. Although mathematics during this era was seen as an independent discipline, due to its useful practical applications, mathematics was still commonly applied in other fields, many mathematicians embraced this fact. For instance, in the late tenth century, the mathematician Abūl-Wafā and the prince Abū Naṣr Manṣur were able to prove theorems regarding plane and spherical geometry that could be used by astronomers and geographers. Such theorems include the laws of sines which solve for sides or angles of a triangle and laws of tangents which were more useful in spherical triangles. Mansur's student al-Bīrūnī was known for mastering the ability to apply mathematic theorems to astronomy and solving for latitude, longitude, or distance between cities in geography. Al-Bīrūnī would go on to produce lots of quality work and become the first person to obtain a simple formula to measure the Earth's radius (Fraiser, 2020)[8]. These developments paved the way for the next generation who would continue branching out into more undiscovered fields, in turn, these studies would be taught to students, fostering a strong educational foundation for the common people. Many new generation mathematicians like Mu'adh al-Jayyani, Abul Wafa, and Omar Khayyam would bloom from this amazing education system and have profound impacts on the world. Al-Jayyani's works reflected the Middle East's 11th-century developments as many of his works were built upon the newly discovered studies like Trigonometry (O'Connor, Robertson, Jayyani, 1999)[11], where he would study unknown arcs and come upon the Law of Sines, a vital formula in modern trigonometry. The Middle East reached such an intellectual peak that many mathematicians namely Omar Khayyam would begin finding flaws and criticizing the original Greek work (Britannica, 2022)[12], which were at the time believed to be completely true. Khayyam specifically found flaws in the book "Euclid's Elements," a book that was the basis of geometry and written by a man known as the father of Geometry. This effort alludes to the intellectual prowess of the Middle East, before mathematicians like Khayyam, most people recognized the Greek works as the ultimate truths due to the Greeks advanced understanding of math which significantly surpassed all other civilizations (save for perhaps India), finding errors within the Greeks works would mean the Middle East had such confidence in their understanding of math that they believed they had surpassed even the Greeks, for Khayyam to find errors within Euclid's work was essentially the passing of the torch from the Greeks to the Middle East, this reflected the societal change as Middle East's position at the top became increasingly harder to dispute as power moved away from the Mediterranean and Europe and into the hands of the Middle East... However, besides challenging his mathematical predecessors, Khayyam was the first person to create the general solution to cubic formulas and built the foundations for the development of non-Euclidean geometry and analytical geometry, he also produced many books on solving algebraic equations (Britannica, 2022)[13], continuing the trend and passing down more knowledge which would lead to even greater future discoveries. Outside of the Middle East, there were also several successful discoveries and inventions notably in India, but the bulk of mathematics contributions still came from the Middle East. This is not to say India had little impact it was quite the opposite, the Indian mathematician Shripati Mishra made strides in probability primarily permutations and combinations. Aryabhata II wrote several books building on other mathematicians' works, discussing numerical mathematics, algebra, and solutions to indeterminate equations. Bhaskara II was one of the most influential mathematicians of this time and wrote a plethora of important treatises many of which were transmitted to Europe and the Middle East. Bhaskara's impressive contributions to mathematics include various forms of geometry (plane, solid, .etc), algebra (notably quadratics, cubics, quartics, and indeterminate equations), and finally Calculus where he conceived differential calculus discovered derivatives, and the differential coefficient (Britannica, 2022)[13]. These contributions were by no means a small feat, but overall the Middle East had become the center of the mathematical world.

12-14th Centuries: Disasters, Decline and a Lack of Advancement

Contrary to the previous centuries, the 12-14th century of the Middle East was characterized by a steep decline from its peak, falling victim to disease, war, and weak rule (Bentley, 2020)[1]. Following the rapid rise of the Middle East, many rulers began to feel content living a life of luxury and leaving their power in the hands of the court. Seljuq warlords from the neighboring Anatolia were able to seize control of the armies and fill the power vacuum ruling the empire through Abbasid puppet rulers. The government fell into corruption and chaos as positions of

power were handed to relatives and friends of Seljuq warlords (Bentley, 2020)[1]. To make matters worse, in the 13th century far east near China, the clans of the Mongolian steppes had banded together uniting for the first time under one leader Genghis Khan who began a rampage across Eurasia, sacking any city that did not surrender to his rule. Soon even Baghdad the golden capital of the Middle East was destroyed by the Mongols, even books were not spared as the Mongols burned down libraries and any books found, destroying centuries of Abbasid research in math and other sciences. Years of conflict and war drained the region of resources, wealth, and lives. Mathematics also reflected this decline as there were few impactful discoveries within three centuries in the Middle East. This was due to the fractured state of the Middle East and the destruction of hundreds of mathematical texts in the libraries of Baghdad. Because of the invasion, the Middle East mainly prioritized military and defense systems over mathematical research which would have less impact on their survival. However, yet another calamity struck the Middle East. In 1346 a new disease known as the bubonic plague was born in China, and through the newly built trade routes like the silk road, the black death was spread across Europe and the Middle East (Bentley, 2020)[1]. This plague had a devastating impact on the Middle East, killing around of the Middle Eastern population. After 3 devastating events, the population and order of the Middle East were in shambles, there was little time to research math as the survival of the people and culture was far more important. As a consequence, the Middle East research fell into disuse. There were almost no prominent mathematic advancements, and much of the works in this time were in the early 12th century before court corruption had reached its height. Works of this time include Al-Samawal giving a definitive answer to what Algebra was "[it is concerned] with operating on unknowns using all the arithmetical tools, in the same way as the arithmetician operates on the known (O'Connor, Robertson, Samawal, 1999)[14]. As of 1130 (the date of this statement) algebra had become a major discipline in Middle Eastern society and the Middle East was widely regarded birthplace of Algebra. A few years following Al-Samawals statement, Sharafeddin Tusi an Iranian mathematician came forward and proposed the idea of a function, although his approach was not explicit many still credit him with being the first to do so, following Omar Khayyam's teachings before him, al-Tusi would also Rufinni Horner's method to numerically approximate the root of a cubic equation and come up with a new way to determine conditions under which certain cubic equations would have two, one, or no solutions, but there are no records for certain of how he discovered this idea. Many modern mathematicians credit Al-Tusi's treatise on equations for introducing algebraic geometry, however, this too has been criticized as like Omar Khayyam before him, Al-Tusi studied equations utilizing curves rather than studying curves utilizing equations. Although Al-Tusi made many mathematical contributions, lots of his work is criticized and his methods are questionable, demonstrating a visible decline in the quality of mathematics which reflected a trend of negligence and sloth within Middle Eastern society during the 12th century. During the 13th century just years before the Mongol invasion, there were a few mathematical advancements but none were nearly as impactful as those during the golden age, such as Kamāl al-Dīn al-Fārisī a Persian optician and mathematician made some contributions to number theory and introduced several new important ideas concerning factorization and combinatorial methods (O'Connor, Robertson, Kamāl al-Dīn al-Fārisī, 1999)[15]. But there was one instance where this was not true, Nasir al-Din al-Tusi who was widely regarded as one of the greatest scholars of medieval Islam, besides being a mathematician Nasir was a highly accomplished philosopher, scientist, and physician. Nasir was one of the only mathematicians who made great strides in mathematic advancement and is often recognized as the creator of Trigonometry as a discipline in its own right. Tusi wrote the "Treatise on the Quadrilateral" a book on spherical trigonometry which entailed a five-volume summary of trigonometry. He was also the first to list 6 distinct cases of the right triangle in spherical trigonometry, and in the year 1250 attempted to develop a new form of non-Euclidean geometry. Nasir was so renowned for his skills and knowledge that during the Mongol invasion, after destroying Alamut castle where Nasir resided, Hulegu Khan, the commander of the army himself was interested in the natural sciences and treated Nasir with utmost respect going as far as to appoint the mathematician as his scientific advisor and later a permanent member of his inner council (Britannica, 2022)[13]. Nonetheless, Nasir was an outlier and the vast majority of mathematicians were unable to make significant contributions. Despite the 3 devastating events, there were some nations untouched by the calamities such as Europe and India. During the 14th century, Western Europe enjoyed a sudden blooming not only in culture and art but also in mathematics under the renaissance. During the Renaissance Europe became less centered around religion and began pursuing activities that improve the human condition (a key aspect of the renaissance), these activities included art, writing, and of course mathematics. The Renaissance gave way to the rise of several European mathematicians, many of whom developed concepts previously discovered in the Middle East and India. Piero Della Francesca an Italian painter wrote books on solid geometry and linear perspective, these were popular mathematic principles in the renaissance as perspective was a key aspect of renaissance art that differed from its medieval counterparts, it was done so with geometric and other mathematic measurements (Carter, 1998)[16]. Luca Pacioli another renaissance mathematician made several contributions to geometric ratios and proportions, a principle in math that fascinated European mathematicians, these ratios and proportions notably the golden ratio were used in almost all renaissance art most famously Raphael's "The School of Athens". These mathematicians would later come to discover advanced

ideas such as Local Motion and the Fibonacci sequence. India which was surprisingly untouched by the events flourished in the 3 century, in the fourteenth-century Madhava worked on a power series for pi and the cosine sine functions, and along with other Kerala school mathematicians, he founded important concepts for Calculus. In the same century Parameshvara a Kerala school mathematician presented a series form of the sine function equivalent to its Taylor series expansion and stated the mean value theorem of differential calculus, Parameshvara was also the first mathematician to give the radius of a circle inscribed with a cyclic quadrilateral. Both India and Western Europe were relatively untouched by major adversity and prospered and discovered an abundance of mathematic concepts while the Middle East and other regions like China which experienced the black death and the Mongol invasion fell into disorder and were unable to produce significant contributions.

15th Century: A Change in Leadership and Discovery

Recovering from the centuries of turmoil, the 15th century Middle East was surprisingly prosperous. Mongols (who ruled the Middle East) used a strategy of moving intellectuals in their empire around to be the most useful, allowing for great inventions that would have taken decades longer to be discovered. This tactic was incredibly useful and led to the invention of weapons, tools, and new technology such as the blast furnace. Moving around intellectuals also benefited the empire as it allowed for the dissemination of knowledge at incredible speeds. Technology like the printing press from China could be taught to a European scholar who upon returning to Europe would teach European smiths how to construct printing presses of their own (Bentley, 2020)[1]. The Ilkhanate (the Middle East under Mongols) remained a great place of learning, perhaps flourishing even more with foreign scholars as they brought along with them their nation's research. A testament to this sharing of knowledge was Jamshid al-Kashi, this astronomer who researched pi and circumscribed circles (Kennedy, 1960)[17], studies which were commonly attributed to India, however, because of the Mongol's connection and centralization, Indian scholars were able to pass on their knowledge to foreign scholars such as Al-Kashi. Mongols were very interested in the natural sciences, specifically, astronomy. They commissioned scholars from across their country to research astronomy. This led to a trend in the Middle East where math was used to design new astronomical instruments such as the astrolabe whose mathematical theory is based on stereographic projections of a sphere, it had been invented previously in late antiquity but the Middle East made extensive developments and improvements (Fraiser, 2020)[8], for example, Syrian Ibn al-Shāṭir's astrolabe which was a mathematical tool that could solve all standard problems of spherical geometry in 5 different ways. Because of this trend, many astronomers were pushed to solve different astronomy-related problems and to do so, they used trigonometry, as a result, highly accurate trigonometry tables were constructed, and completely new trigonometry theorems were developed, leading to the development of Trigonometry as an independent mathematical discipline (Fraiser, 2020)[8]. Trigonometry reached quite a peak in this period, as many Mongolian khans themselves were interested in this new discipline an example would be Ulugh Beg the grandson of Mongol conqueror Timur or better known as Tamerlane. Beg was himself a great mathematician and astronomer, his achievements include constructing sine and tangent tables for every minute of an arc, he also became the patron of Jamshid Al Kashi, one of the greatest mathematicians and astronomers of the Islamic world. Besides his achievements stated previously, Al Kashi's "The Reckoners Key" summarized almost all of the arithmetic in his time and even included sections on algebra and practical geometry (Samplonius, al-Kāshī, 2022)[18]. His works were of such quality that Ulugh Beg described Al Kashi's works as "known among the famous of the world." This description illustrates a rise in social status and acknowledgment of mathematic accomplishments Ulugh Beg's patronage of mathematics draws parallels with 10th and 11th-century Abbasid governments' funding of research which brought about a plethora of discoveries in the world of mathematics. The booming international trade under the Ilkhanids, which stretched from the Mediterranean to China, produced wealthy merchants who financially supported scholars like Al-Kashi. Their patronage allowed mathematicians to develop algebra further and publish mathematical treatises solving commercial problems involving currency exchange. Rising tax revenues for the Ilkhanid state also provided funding for advanced projects such as the renowned Maragha observatory established by Nasir al-Din al-Tusi under Hulagu Khan's sponsorship. Al-Tusi spearheaded innovations in trigonometry and astronomical tables that built upon the Greek mathematical tradition. Meanwhile, the construction and management of expanding urban infrastructures like those in the new Ilkhanid capital of Tabriz employed mathematicians, including al-Qazvini who wrote extensively on applied geometry, advancing the field and its applications to engineering. Overall this reflected a trend in the Illkhanate of prosperity rather than conquest as Mongol khans settled down and began focusing on building up their regions, many trade routes were rebuilt and patrolled which gave rise to economic productivity and dissemination of knowledge like never before. Nevertheless, the Middle East was flourishing with or without foreign influence, and intellectuals like Ibn al-Banna and al-Qalasadi continued making mathematics more accessible and simpler for common people by developing symbol notation for algebra (O'Connor, Robertson, Qalasadi, 1999)[15], this development reflected the social advancement of more power and rights entering the hands of the common people. This era could be

seen as a golden era in its own right, and many of the discoveries were vital to unlocking the information we know today. But, although the Middle East was still a world power to be reckoned with, Europe's rise became increasingly obvious to neighboring countries, riding the wave of the renaissance, the dissemination of knowledge spread important knowledge of ships, gunpowder, and mathematics to European shores. Just as the Abbasids had translated Greek texts, Europeans would translate many Arabic texts. Additionally, Europe adopted Hindu Arabic numerals and public schools (Du Sautory, 2008)[6]. Following the 15th century, the Middle East would pass the torch to the western world who in the following centuries would invent and discover like never before, transforming the world through the enlightenment and industrial revolution (Bentley & Hill, 2020)[1].

Conclusion

In sum, the development of mathematics reflected the advancement of Middle Eastern society from the 8th-15th century AD quite accurately. During the rise of the Middle East (8th to 9th century) the Middle East did not make huge contributions to mathematics but instead learned and took in information from more advanced kingdoms which reflected the characteristics of a still developing society, mathematics was also used for practical purposes as there were not enough resources for many to study mathematics as an independent discipline. This all changed in the Golden Age of the Middle East (10th to the 11th century) where there was an abundance of resources and wealth that allowed the government to fund many projects for the study of mathematics and other fields, this in turn led to an explosion of discoveries from Middle Eastern mathematicians and other scholars which propelled the Middle East from a rising nation to an advanced empire full of elite scholars of the highest caliber. When the Middle East fell into disarray following a corrupt government, the Mongol invasion, and the bubonic plague during the eleventh to the twelfth century, the lack of notable mathematic contributions was able to reflect this decline, and other regions that did not experience any major tragedies like India and Europe were able to prosper in the world of mathematics, and in Europe's case begin their Golden Age. Finally, in the fifteenth century under new rulers, the Middle East once again experienced a flourishing golden age which was alluded to by their impressive progress in Trigonometry. However, they would inevitably be overtaken by rising Europe riding the wave of the Renaissance.

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