Exploring Cosmic Microwave Background Anisotropies and Polarization through Simulated Analysis

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The cosmic microwave background (CMB) radiation, a relic of the early universe, holds crucial insights into the origins and evolution of our cosmos. In this paper, we present a detailed analysis of CMB temperature anisotropies and polarization, employing innovative methodologies to probe the inflationary epoch and explore crucial cosmological phenomena. The analysis centers on understanding the properties of CMB temperature fluctuations, which reveal density fluctuations in the early universe and serve as seeds for the formation of cosmic structures. Additionally, the investigation distinguishes between E-modes and B-modes in CMB polarization, with the latter holding potential implications for detecting primordial gravitational waves generated during inflation. Simulations of CMB maps are performed, accounting for various sources of noise, including point sources, the Sunyaev-Zel'dovich effect, atmospheric, and instrumental noise. Data processing techniques, such as filtering and corrections, are employed to obtain unbiased and precise measurements of the underlying cosmological signal. The results of this analysis contribute to the understanding of the early universe and provide valuable insights into the validity of the inflationary theory. This paper finds properties of CMB temperature fluctuations and the distinct features of polarization modes. Through this simulated analysis, we can pave the way for future observational studies. While the data used in this study is synthetic, it serves as a valuable tool to refine our analysis techniques and prepare for real-world observational endeavors. This research provides contributions to the field of cosmological studies, enabling further exploration and deeper understanding of the fundamental nature of our cosmos.

Introduction

The cosmic microwave background presents a treasure trove of information that has revolutionized our understanding of cosmology. In this research paper, we undertake an analysis of CMB temperature anisotropies and polarization, aiming to unlock crucial insights into the early universe and shed light on the validity of the inflationary theory.

The CMB, discovered in 1965, serves as a pristine snapshot of the universe when it was just 380,000 years old, unveiling the conditions that prevailed during its infancy [1]. One of the most remarkable aspects of the CMB is its remarkable uniformity, with tiny temperature fluctuations revealing density variations in the early cosmos [1]. These fluctuations, which are small variations in the CMB, play a crucial role in helping us understand how cosmic structures, such as galaxies and galaxy clusters, formed and how the large-scale structure of the universe developed over time [1]. The inflationary theory, introduced in the early 1980s, proposes a rapid and exponential expansion in the universe's earliest moments, offering a solution to cosmological problems [2]. During this brief inflationary epoch, quantum fluctuations were amplified, leading to the formation of seeds for galaxies and galaxy clusters [2]. The detection of gravitational waves imprinted on the CMB polarization could offer direct evidence of this inflationary expansion, solidifying the theory's foundation [3].

The primary goal of this study is to conduct a simulated analysis of CMB temperature anisotropies and polarization. Specifically, we aim to investigate the potential detection of primordial gravitational waves through the analysis of B-modes in CMB polarization. To achieve this, we perform simulations of CMB maps, considering various sources of noise that can affect our measurements. By employing data processing techniques such as filtering and corrections, our aim is to obtain precise and unbiased measurements of the underlying cosmological signal. Although data is simulated, a successful result will pave the way for future research in analyzing B-modes through improving analysis techniques for noise reduction and refined measurements.

Background

Inflation

Inflation is a theory that proposes the rapid exponential expansion of space in the early universe. This period of time, referred to as the inflationary epoch, is believed to have lasted $10^(-36)$ seconds to between $10^(-33)$ and $10^(-32)$ seconds after the Big Bang [2]. The rapid expansion creates gravitational waves which imprint a particular polarization pattern in the cosmic microwave background, remnant radiation from the Big Bang.

Detecting this pattern would leave us with a remnant of the big bang, transforming our fundamental understanding of physics. The strength of the signal detected from the gravitational waves would allow scientists to directly measure the rate at which the universe expanded during inflation. Inflation also allows us to observe quantum fluctuations created in the early universe, as expansion during the inflationary epoch magnified the quantum fluctuations, leaving imprints in the cosmic microwave background. The theory was introduced in 1981 to solve several problems in cosmology, mainly the horizon problem, flatness problem, and the magnetic monopole problem[2].

Horizon Problem

The horizon problem results from the uniformity of a background temperature in the universe known as the cosmic microwave background (CMB)[4]. This uniformity is puzzling because it implies that distant regions of the universe must somehow be in contact with each other; however, distant regions of the universe today are so far apart that the CMB light travel time between them exceeds the age of the universe, which means they must have been in causal contact in the past4. Inflation proposes that the universe underwent a brief period of exponential expansion, which would have stretched the observable universe from an incredibly tiny region to a size much larger than the current observable universe. Based on the inflation theory, these distant regions would have been much closer together before the Big Bang, allowing for the resulting thermal equilibrium today[4].

Flatness problem

The term "flatness" refers to the geometry of the universe, specifically the spatial curvature. The geometry of the universe can take three forms: flat, open, or closed. The flatness problem arises from the fact that the observed density of matter and energy in the universe from the CMB is incredibly close to the critical density required for the universe to be exactly flat[5]. Over cosmic time, any deviation from the critical density should increase significantly. If the universe had a higher density, it would imply a closed universe with positive curvature. If the density was lower, the universe would be open with negative curvature. As a result, the universe should naturally evolve away from flatness as it expands, contradicting the observed density of the universe today[5]. The inflation theory offers a solution to the flatness problem. During the rapid expansion of inflation, the geometry of the universe is stretched and smoothed out[3]. The matter and energy of the universe is spread out during the expansion. Inflationary expansion causes the cosmic density parameter to converge towards 1, regardless of its initial value. This means that inflation can naturally explain the observed flatness of the universe by providing a mechanism for the early universe to become extremely close to critical density[3].

Magnetic Monopole Problem

A magnetic monopole is a hypothetical particle that carries a magnetic charge similar to how an electron carries an electric charge[6]. While individual magnetic poles (north and south) are observed in everyday magnets, isolated magnetic monopoles have not been detected experimentally[6]. According to the principles of grand unified theories, at extremely high energies, the electromagnetic force, weak nuclear force, and strong nuclear force can merge into a single unified force . If grand unified theories are correct and the universe underwent a phase transition in the early moments after the Big Bang, magnetic monopoles would have been produced abundantly[6]. However, the observed absence of magnetic monopoles in the universe is in stark contradiction to the predictions of these theories[6]. Inflation provides a mechanism for explaining the scarcity of magnetic monopoles by pushing them outside the observable horizon and making them unobservable in our part of the universe[3]. Inflationary theory postulates a period of rapid expansion in the early universe, which can significantly dilute the abundance of magnetic monopoles and other relics from the grand unified phase transition. The exponential expansion during inflation would cause the regions of the universe that contain magnetic monopoles to become extremely diluted, making them extremely rare or even absent in observable regions of the universe[3]. While they might still exist in remote regions beyond our horizon, they would be too far away for us to detect.

Cosmic Microwave Background

The cosmic microwave background (CMB) is a faint radiation permeating throughout the universe, and is a record of the universe from approximately 380,000 years after the Big Bang[1]. It manifests as a nearly uniform glow, with an average temperature of approximately 2.7 Kelvin. Despite its uniformity, small temperature variations exist, reflecting density fluctuations in the early universe that later grew into the galaxies and large-scale structures we observe today[1].

In the early universe, immediately after the Big Bang, the universe was incredibly hot and dense. It was in a state of a plasma, consisting of a soup of charged particles, mainly protons and electrons[1]. Photons, the particles of light, were continuously scattered by these charged particles, preventing them from traveling freely through space. As the universe expanded and cooled down to a critical temperature of approximately 3,000 Kelvin, recombination occurred, allowing protons and electrons to combine and form neutral hydrogen atoms[1]. The neutral hydrogen atoms no longer interacted with photons, allowing them to travel through space. This process is known as decoupling[1].

These photons, which were originally very energetic, have undergone redshifting as the universe expanded. This redshift stretched the wavelength of the photons, shifting them from the high-energy range (such as X-rays or gamma rays) into the

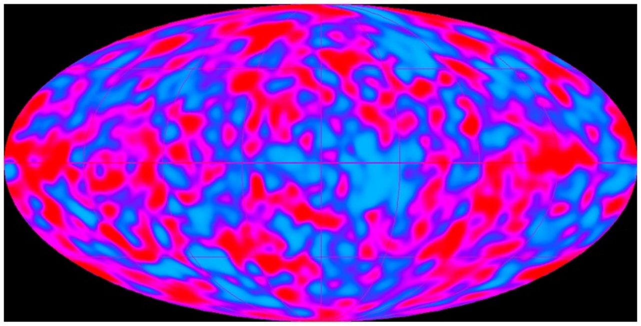

Fig. 1 COBE's CMB temperature anisotropy map. Temperature fluctuations are represented by color variations in the map.

microwave region of the electromagnetic spectrum today [1]. The CMB exhibits the characteristic thermal blackbody spectrum, known as the Planck spectrum. Since scientists understand the behavior of blackbodies, it allows them to deeply analyze the features of the CMB.

Analysis of the CMB allows access to important information regarding the early universe, including whether the inflationary event actually occurred. Rapid expansion during inflation would have formed gravitational waves, leaving a unique polarization pattern within the CMB, specifically the B-mode polarization signal. Analyzing this pattern would provide scientists the chance to directly measure the inflationary event and provide strong evidence to support the inflation theory.

The first full-sky map of the CMB was created through NASA's Cosmic Background Explorer (COBE) mission launched in 1989 [1]. The image supported predictions made by the Big Bang theory while also providing information on cosmic structure. In 2003, the Wilkinson Microwave Anisotropy Probe (WMAP) provided a more detailed map, shedding light on the composition, age, and geometry of the universe [1]. WMAP estimated the age of the universe at 13.7 billion years and revealed that the oldest stars started shining 200 million years after the Big Bang [1]. More recently, the European Space Agency's Planck space telescope released the currently highest precision picture of the CMB from a satellite mission. It confirmed information from WMAP and provided more proof behind the existence of dark matter and dark energy [1].

Temperature anisotropies

Temperature anisotropies refer to the fluctuations in temperature across different regions of the CMB. While the CMB has a nearly uniform temperature of about 2.7 Kelvin, these anisotropies represent small deviations from this temperature [7]. Since the anisotropies are results of the imprints left by the physical processes and conditions present during the CMB's formation, they provide information about the structure of the early universe [7].

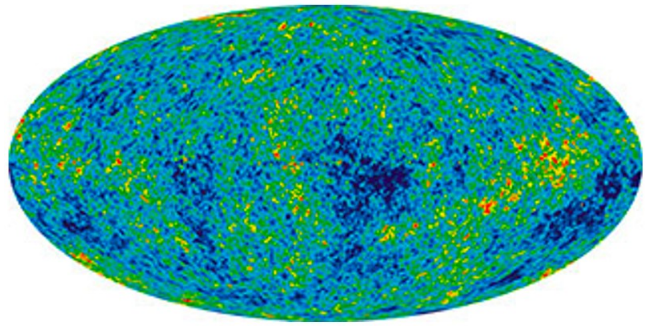

Fig. 2 WMAP's CMB temperature anisotropy map. Temperature fluctuations are represented by color variations in the map, in higher resolution than before.

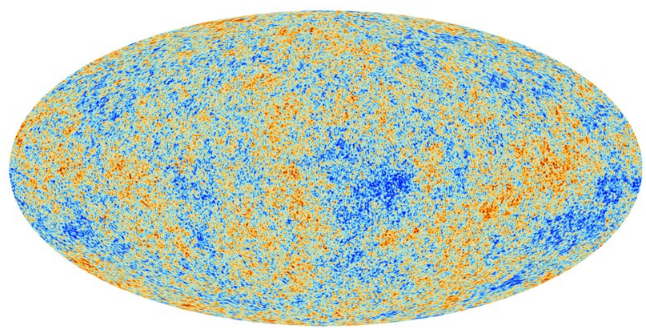

Fig. 3 Planck's CMB temperature anisotropy map. Temperature fluctuations are represented by color variations in the map, with further improved resolution.

The temperature anisotropies in the CMB are directly related to the density fluctuations in the early universe. These fluctuations serve as the seeds for the formation of cosmic structures, such as galaxies and galaxy clusters, through gravitational collapse[7]. Regions with slightly higher density had a stronger gravitational pull, attracting more matter towards them. As photons passed through these regions, they experienced a gravitational redshift, losing energy and appearing cooler in the CMB. On the other hand, underdense regions had weaker gravitational attraction, resulting in photons retaining more energy and appearing warmer. Therefore, the density fluctuations in the early universe affected the paths of CMB photons, leading to temperature anisotropies[7].

Polarization in the CMB

Polarization refers to the alignment of light perpendicular to its direction of propagation. For the CMB, there are two main types of polarization anisotropies: E-modes and B-modes[3]. Density fluctuations in the early universe induced variations in the polarization of the CMB photons. The gravitational potential associated with the density fluctuations caused the photons to experience a slight stretching or compression as they traversed the universe. This stretching or compression led to a preferred direction of polarization for the photons, resulting in the E-mode polarization pattern, which can inform us about the way structure (galaxies) formed. Since density fluctuations only create polarization patterns of this specific type, E-modes end up being the dominant polarization type in the CMB[3]. E-mode patterns typically exhibit a smoother and more symmetric distribution across a CMB map. They are driven by scalar (density) perturbations and are not affected by gravitational waves.

Unlike the E-mode polarization, the B-mode polarization is primarily produced by primordial gravitational waves[3]. During the inflationary period of the early universe, quantum fluctuations in the gravitational field generated gravitational waves, leaving an imprint on the CMB radiation and generating the B-mode polarization pattern. Primordial B-modes have not yet been observed, and detecting them proves challenging. Polarized emissions from sources such as galactic dust can contaminate the B-mode signal. These emissions can have polarized patterns that mimic the expected B-mode signal from gravitational waves, so it is important to determine whether signals originate from the early universe or more locally. However, if detected, B-mode polarization would be crucial in proving the existence of primordial gravitational waves and confirming the inflation theory[3]. B-mode patterns are often more complex and have a characteristic "swirl" pattern. Unlike E-mode, B-mode patterns exhibit a clear sensitivity to gravitational wave perturbations.

CMB temperature power spectrum

The CMB temperature power spectrum is a fundamental tool in cosmology that describes the statistical properties of temperature anisotropies in the CMB radiation. The CMB temperature power spectrum is obtained by analyzing these temperature anisotropies across different angular scales[8]. Angular scales correspond to different sizes in the sky, ranging from large angular scales (low multipoles) to small angular scales (high multipoles). The multipoles can be thought of as spherical harmonics, which mathematically represent the different patterns of fluctuations in the sky[9].

The power spectrum is usually plotted as a function of multipoles on the x-axis and the amplitude of fluctuations on the y-axis. The power spectrum quantifies the amplitude of fluctuations at each multipole. It represents the statistical distribution of power across different angular scales[7]. In this case, power spectrums of temperature anisotropies are used, so peaks and troughs in the power spectrum represent variations of temperature in the CMB. The power spectrum quantifies how the amplitude of temperature fluctuations varies with angular scales in the sky.

Limitations of simulation

While simulations strive to capture the essence of astrophysical processes, it's important to note that certain complexities may be simplified due to computational constraints. The fidelity of simulations in replicating the intricacies of the universe is subject to inherent limitations. Simulations often assume ideal calibration, and uncertainties present in real-world observational instruments may not be fully represented. Simulated data is taken from the NASA LAMBDA CAMB interface. The interface operates based on specific cosmological parameters, and while it provides a controlled environment for simulations, deviations from real-world observational conditions are inevitable. The simplicity of input parameters may not fully encapsulate the intricacies of the universe, potentially leading to discrepancies between simulated data and actual observational outcomes. However, the interface provides easy access to data, allowing explorations of a wide range of cosmological scenarios. Simulations are still meant to represent reality to the best of its ability, and provides convenience for testing analysis.

Methods

Angular Temperature Power Spectrum

A CMB angular power spectrum contains the majority of information content in the CMB. The spectrum represents the amplitude of temperature fluctuations as a function of multipoles. Using the NASA LAMBDA CAMB interface we can generate these spectra by giving it a set of simulated cosmological parameters. CAMB (Code for Anisotropies in the Mi-

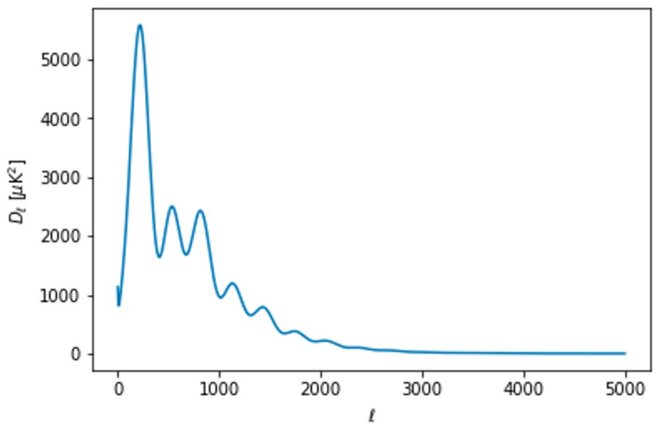

Fig. 4 Plot of the angular power spectrum of the CMB temperature anisotropies. The x-axis represents the multipole moment l. For example, $l = 10$ would represent about 10 degrees on the sky. The y-axis represents the amplitude of temperature fluctuations in the CMB.

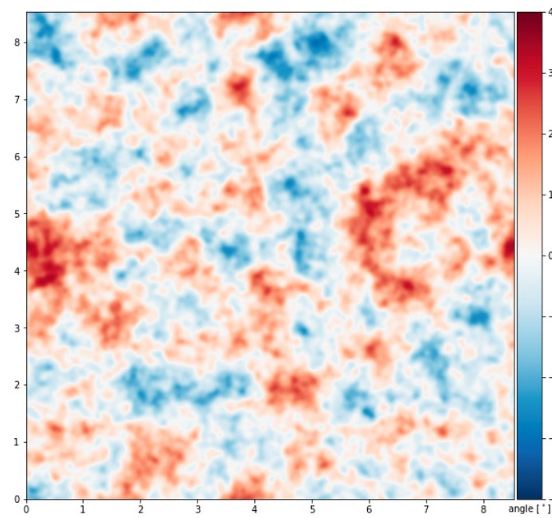

Fig. 5 A contour plot showing the simulated map of the CMB sky. The map represents temperature fluctuations within this portion of the CMB. Brighter regions in the contour plot indicate higher temperature fluctuations, while darker regions represent lower fluctuations. Figure 5 shows an ideal result of a CMB signal. However, the simulated data does not accurately represent various factors, including point sources, the Sunyev-Zeldovich (SZ) effect, atmospheric noise, and instrumental noise. To account for these sources, we can generate maps representing them and combine them with the simulated CMB signal.

crowave Background) is a software package used in cosmology to compute the theoretical predictions for the anisotropies of the cosmic microwave background (CMB)[10]. It is a tool for modeling anisotropies in the early universe, allowing researchers to generate theoretical predictions based on different cosmological parameters. It includes code for calculating lensing, galaxy count, matter power spectra, and transfer functions. The simulated data, along with the following data analysis techniques, are referenced from the Github Repository

Temperature Anisotropy Map

A simulated map of the CMB can be generated for closer analysis, using the above angular spectrum. The map considers a small patch of the sky ($\sim 10^\circ \times 10^\circ$) in a flat-sky approximation. As a result, l is replaced with $k = \sqrt{x^2 + y^2}$ to have x and y coordinates.

To generate a CMB map in the flat-sky approximation, we have to:

1. Generate a 2D power spectrum by rotating the CMB power spectrum about the axis in polar coordinates.
2. Generate a Gaussian random map.
3. Multiply the maps from 1 and 2.
4. Fourier transform to convert the result from harmonic space to real space, creating the simulated map.

Point Source Map

In CMB maps, point sources can arise from various astrophysical objects. For the purpose of simulated CMB maps, a faint population of sources is considered, and its brightness follows a Poisson distribution. This accounts for the presence of numerous dim sources in the CMB maps. In addition to the faint sources, a smaller number of very bright sources is included. These bright sources are characterized by an exponentially decreasing source count. This aspect of the simulation mimics the presence of a limited number of exceptionally bright sources observed in real CMB maps. The parameters governing the behavior of these simulated sources have been manually chosen to visually resemble real CMB maps at a specific frequency (150 GHz).

Sunyev-Zeldovich (SZ) map

Clusters of galaxies cause subtle distortions in CMB maps. These distortions arise due to the interaction of CMB photons with hot electrons in the ionized gas present within the gravitational potential well of the cluster, or inverse Compton scattering. As a result, the photons gain energy, leading to a distinct effect in the CMB signal known as the SZ effect. At frequencies below the "null" at 220 GHz, the SZ effect causes a decrease of power, while at higher frequencies, there is an increase in power. Importantly, the SZ effect is redshift independent, making it a useful tool for detecting clusters of galaxies regardless of their distance from us.

For the simulation, clusters are treated as having a brightness

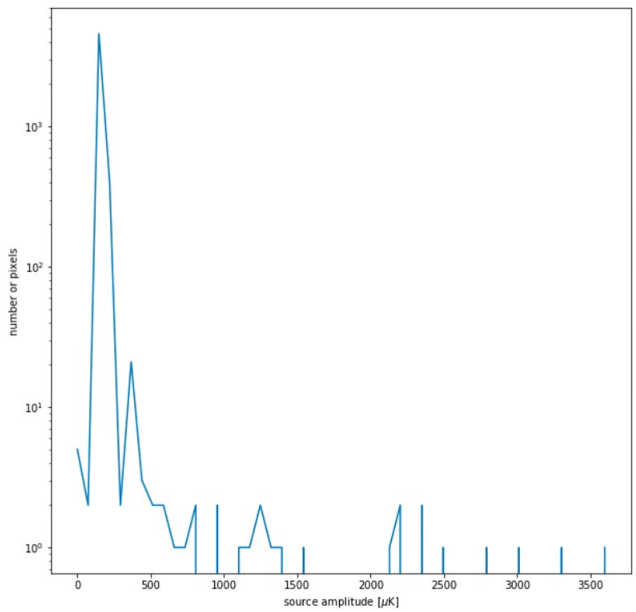

Fig. 6 A histogram of the brightness of the pixels in the simulated source map. The x-axis represents brightness values, while the y-axis shows the frequency of pixels at each brightness level.

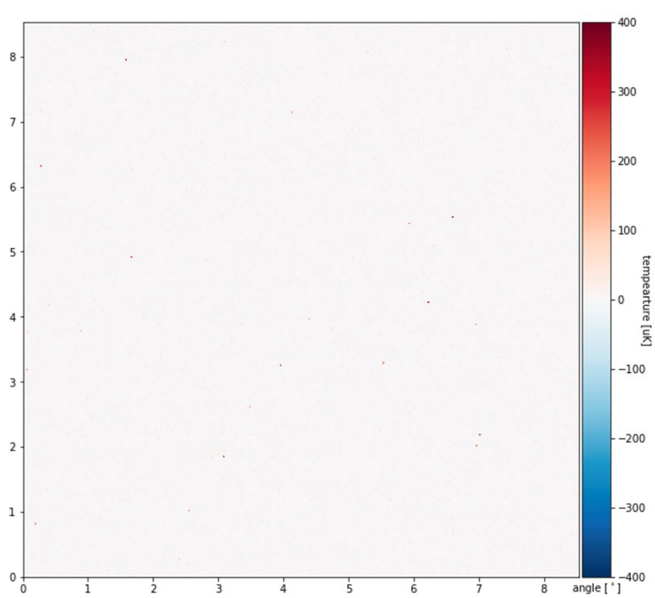

Fig. 7 The simulated point source map. The map visually represents the distribution of point sources across the CMB sky.

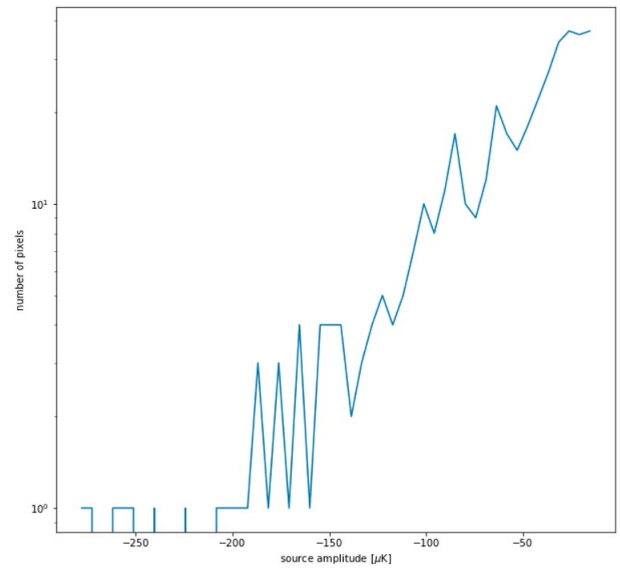

Fig. 8 A histogram of the SZ-decrements obtained from the simulated SZ cluster map. The histogram represents the distribution of SZ-decrement values in the map. The x-axis represents the SZ-decrement values, while the y-axis shows the frequency of occurrences for each SZ-decrement value.

"beta profile," and with each cluster having an identical angular size. For more accurate simulations, a range of cluster sizes, a distribution of cluster shapes with more accurate profiles, and a choice of the number of clusters as a function of mass and redshift should be used to match measurements of the cluster mass function.

Sky Map Convolved with a Beam

Telescopes experience finite resolution effects due to diffraction. To counteract this, we create a Gaussian beam pattern, which approximates the telescope's diffraction pattern.

Noise Map

Ground based CMB instruments are affected by various types of noise, including white noise, atmospheric noise, and detector noise. It is important to model and emulate these noise components for more accurate analysis of the CMB. Both atmospheric and detector noise manifest as correlated noise in map space. However, these noise components are nearly uncorrelated in Fourier space. This means that their effects in map space can be separately modeled and analyzed.

To account for noise in the simulation, we choose a white noise level of 10 μK-arcmin resembling current deep maps from CMB experiments. The atmospheric noise is modeled using a 2D spectrum similar to the CMB, leading to similar patterns in

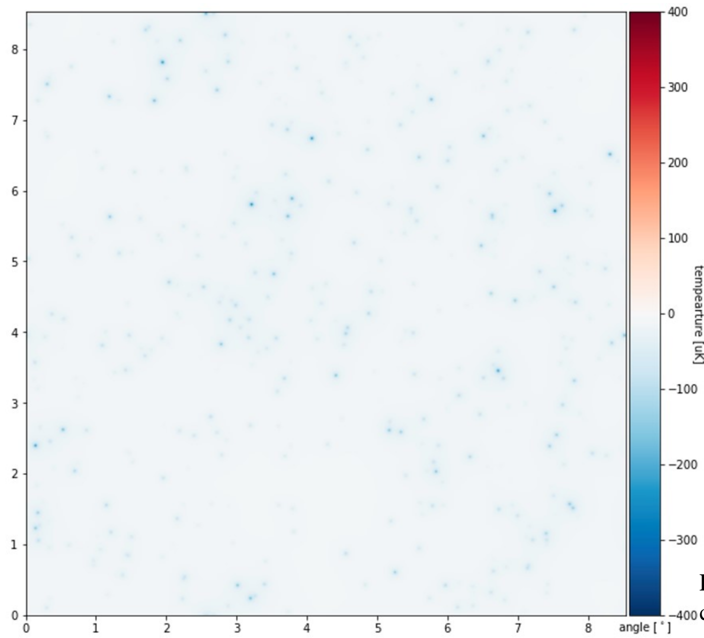

Fig. 9 The simulated SZ cluster map at 150 GHz. The map visually presents the SZ effect caused by clusters of galaxies on the CMB. The brightness of the map represents the SZ-decrement value at that location.

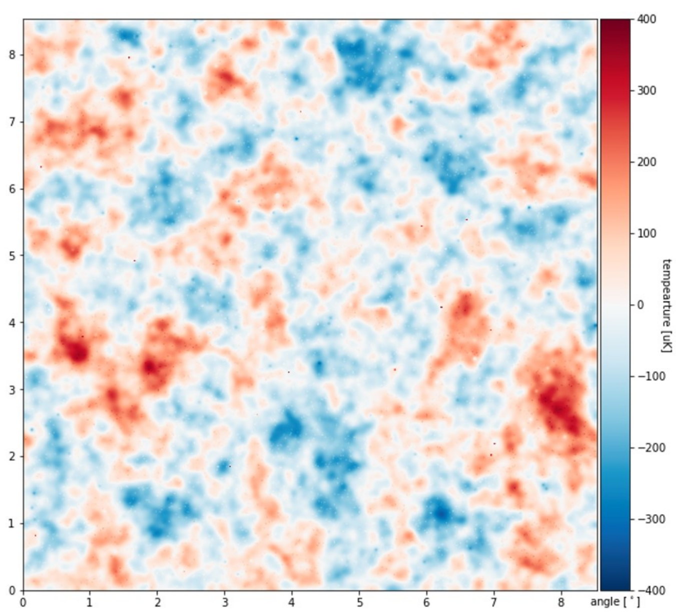

Fig. 10 The full sky map, a simulated map that combines the CMB, point source, and SZ signals.

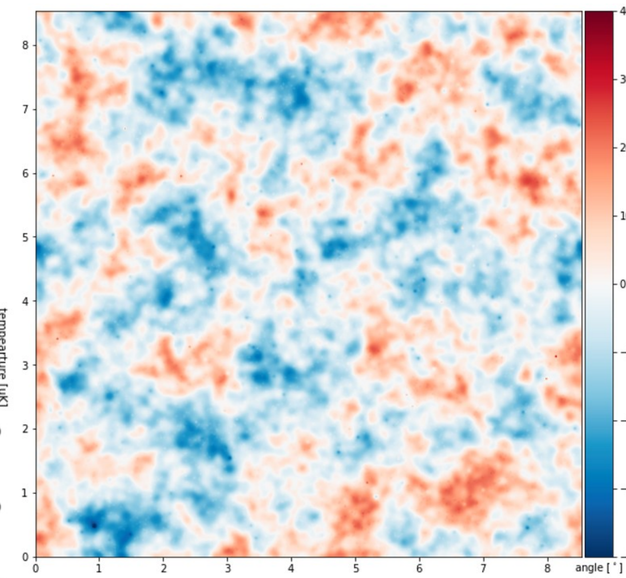

Fig. 11 A plot including the CMB, point source, and SZ signals and is convolved with an instrumental beam.

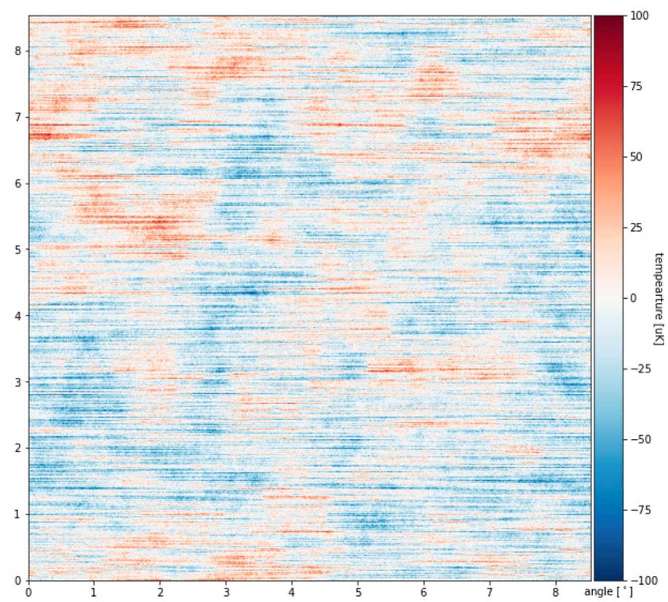

Fig. 12 A plot displaying a realization of instrumental and atmospheric noise in the simulated data. The atmospheric noise component exhibits similarities with the CMB signal due to sharing a similar 2D spectrum.

the maps.

Filtering

Due to multiple sources of noise affecting the raw CMB signal, filtering can be used to target these sources to reduce noise.

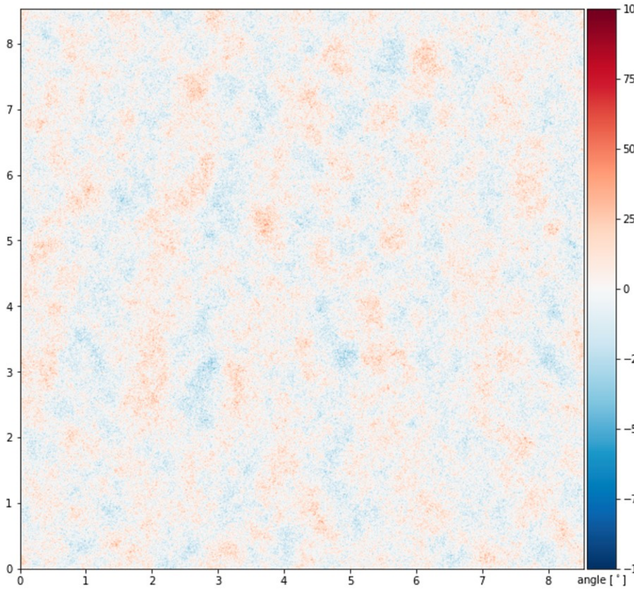

Fig. 13 A plot displaying filtered frequency-dependent noise.

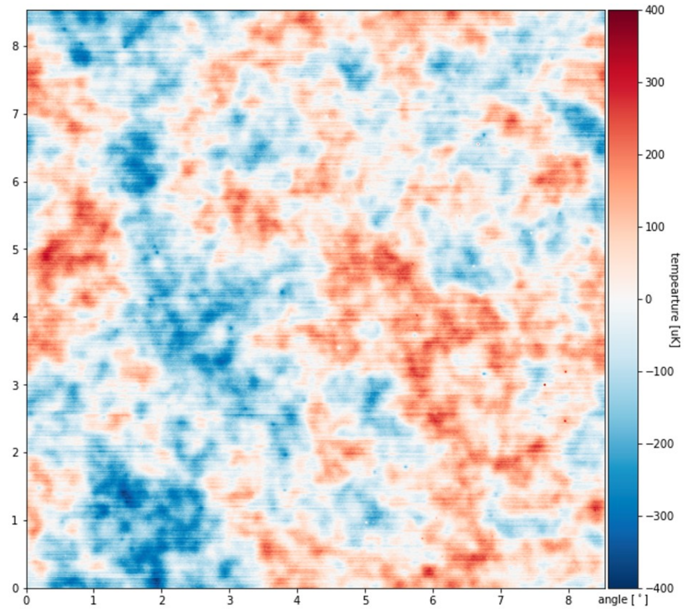

Fig. 14 A completed simulated CMB map, created by adding the simulated sky map convolved with the beam to the simulated noise maps.

Filtering allows a cleaner signal for better analysis of the CMB.

For maps with frequency-dependent noise, we can apply filtering techniques to "whiten" the noise spectrum. The purpose of such filtering is to modify the noise distribution, making it more uniform and independent of frequency. For both atmospheric and detector noise, a high-pass filter would be an effective choice. This type of filter allows shorter wavelength modes (higher-frequency components) to remain unchanged while damping or eliminating longer wavelength modes (lower-frequency components) below a certain cutoff.

Map Analysis

Edge effects

In the context of CMB analysis, a 2D Fast Fourier Transform (FFT) is used to transform a signal from real space (position or temperature on the sky) to Fourier space (frequency or angular scale). This can be used to convert the temperature anisotropy map from its pixel representation to the angular power spectrum. The FFT transforms the signal into a frequency domain, revealing the contribution of different frequencies to the overall signal. Peaks in the frequency domain correspond to dominant frequencies in the original signal.

Before taking a 2D Fast Fourier Transform to compute a power spectrum, we must apodize the maps to eliminate edge effects. These edge effects result from the Fourier transform treating the map as having periodic boundaries, which can lead to spurious signals if the values on opposite edges of the map do not match.

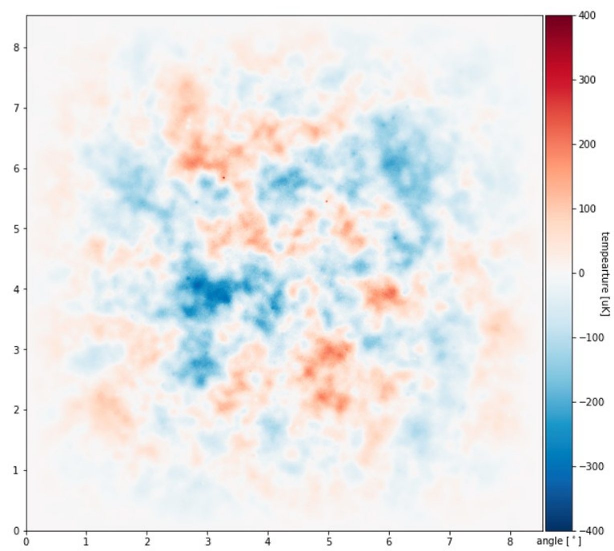

Fig. 15 The simulated map with a cosine window applied to eliminate edge effects.

We can use a cosine window for apodization. The cosine window smoothly tapers off the signal to zero towards the edges, effectively reducing the impact of edge effects during the Fourier transform.

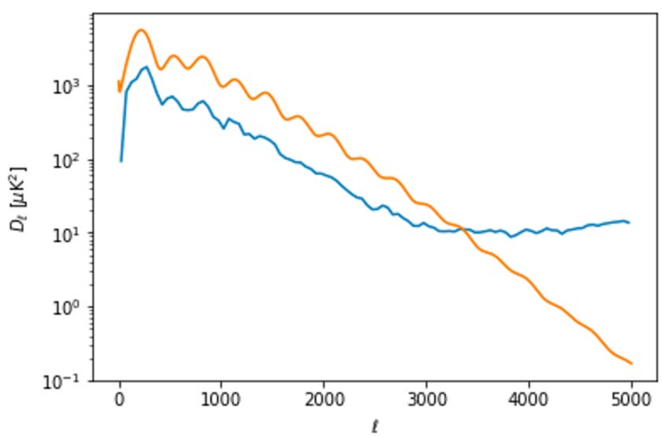

Fig. 16 A plot representing the input power spectrum (orange) and the estimated naive power spectrum (blue). They do not match due to instrumental noise, SZ and point source signals, suppression from the beam and the apodization.

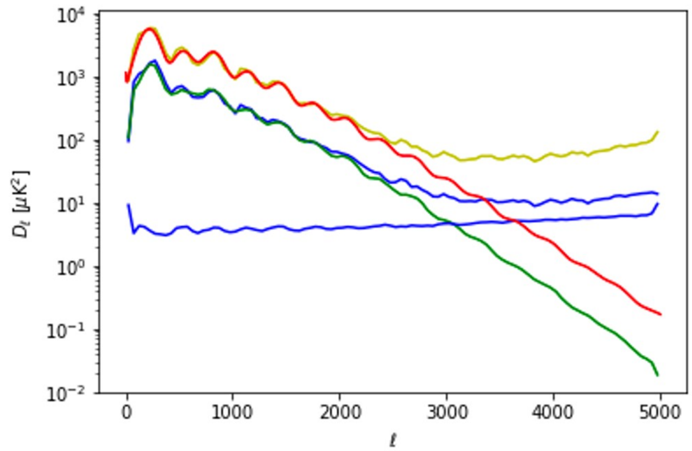

Fig. 17 The estimate of the CMB power spectrum after correcting for the multiplicative bias (transfer function) is shown in yellow. The transfer function is represented by the lower blue curve. In addition, the plot shows the input CMB power spectrum (red), the average of the signal-only simulations (green), and the naive power spectrum (blue, upper).

Naive Power Spectrum

We want to compute a naive power spectrum and compare it to the input power spectrum used in the simulations. The power spectrum computation involves:

- Performing a 2D FFT on the map, converting the map from real space to Fourier space.
- Taking the absolute value squared of the Fourier map.
- Averaging the signal in annular bins of $k = \sqrt{x^2 + y^2}$. These bins are converted to l with the $l = k \times 2\pi$ scaling factor appropriate for the flat sky approximation.

Correcting biases

Addressing noise bias is important since the CMB signal is extremely faint, and any additional noise, whether instrumental, atmospheric, or due to other astrophysical sources, can distort or obscure the true signal. To obtain an unbiased estimate of the true underlying power spectrum in our simulated map, we must correct biases in the naive power spectrum. The naive spectrum P_{measured} is related to the true underlying spectrum P_{true} through additive noise N, and transfer function T which accounts for the effects of the instrument's beam and filtering. Mathematically, the relation is:

$$P_{\text{measured}} = T \times P_{\text{true}} + N$$

To recover the true power spectrum, we use Monte Carlo techniques. Simulations can be used to calibrate T and N, allowing us to correct the naive measurement using algebra:

$$P_{\text{true}} = \frac{P_{\text{measured}} - N}{T}$$

Calibrating the transfer function T: The transfer function is calculated by:

- Generating sky simulations with a known power spectrum, and modeling the instrument's transfer function and post-processing effects. To ensure that the simulations only capture the impact of the transfer function, we set the noise level to zero.
- Calculating the naive power spectrum from each simulation.
- Running a large number of simulations to reduce numerical noise and improve accuracy.
- Averaging the naive power spectra to an average signal-only spectrum, then dividing the true power spectrum by this average signal-only spectrum to estimate the transfer function.

Calibrating the noise bias

The noise bias can be computed by generating noise only simulations, calculating the noise power spectrum from each simulation, and finding the average power spectrum.

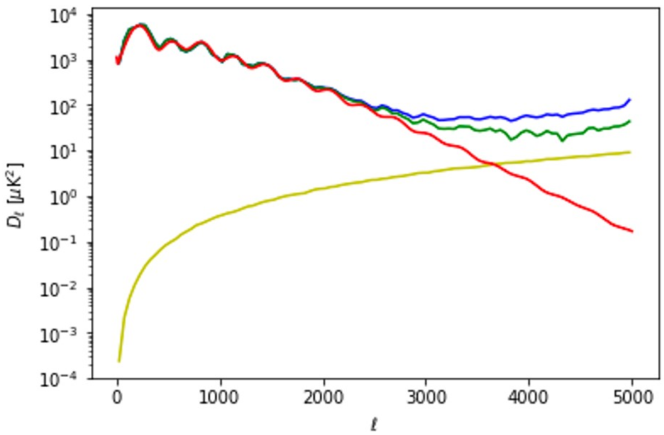

Fig. 18 A plot of the unbiased estimate of the power spectrum is shown in green, which includes corrections for both the noise bias and the transfer function. The plot also includes the estimate for noise-only additive bias (yellow), the power spectrum accounting for only the multiplicative bias (blue), and the underlying power spectrum used to generate the map (red).

Quantifying the error bars

To compute the error bars on the unbiased estimate of the power spectrum, we can generate simulations including signal and instrumental noise, compute the naive power spectrum from each simulation, taking the root mean square (RMS) of the results, subtracting the noise bias and accounting for the transfer function.

Conclusion

This study presents an analysis of cosmic microwave background (CMB) temperature anisotropies and polarization through simulations that incorporate realistic noise to simulate real-life observations. Through simulations of CMB maps with added noise to mimic real observational conditions, we obtained precise measurements of the underlying cosmological signal. The incorporation of realistic noise allowed us to assess the impact of various sources of interference, such as point sources, the Sunyaev-Zel'dovich effect, atmospheric, and instrumental noise, on the accuracy of our analysis. By conducting a simulated analysis with added noise, we have taken a practical step towards understanding the complexities of CMB observations in realistic scenarios. The results of this research contribute to the broader field of cosmological studies, providing essential insights for future observational studies that will grapple with the complexities of real data. Future analysis should be done to account for more variations in simulated data, such as noise or other filtering techniques. Furthermore, comparison with possible future empirical data should be done to confirm the accuracy of the simulation. However, the simulated analysis presented here serves as a foundation for further exploration and paves the way for forthcoming observational studies.

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