Constraints on Primordial Black Holes as Dark Matter

Xinxin Fang

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The identity of dark matter is an outstanding problem in astrophysics. In this paper, we discuss an observational method that can potentially reveal dark matter's nature. Recent progress has expanded the list of possible candidates of dark matter. One possible candidate is discussed: primordial black holes (PBHs). In addition to introducing the formation of PBHs, we look at constraints on primordial black holes as fraction of dark matter that can be based on a variety of observational evidence.

Introduction

In the study of cosmology, or the evolution and dynamics of our universe, a large research program has developed to determine the matter density and composition of the universe throughout time. The purpose for this is to find what the universe comprises and how it came to be like it is today. Some types of matter, such as stars, are detectable since they broadcast photons, or particles of light that can be directly observed. However, most matter is undetectable optically or via instruments that utilize wavelengths beyond the visible light range in the electromagnetic spectrum. We've discovered empirical evidence that matter in the universe is mostly nonbaryonic, which does not interact with any electromagnetic radiation. Because of this property, this matter is called *dark matter*. Dark matter is a nonluminous matter that doesn't affect any type of electromagnetic radiation and composes 85% of matter in the universe. With the predominant scientific view that dark matter consists of fundamental particles, it is surprising that the fundamental particles of much of the universe is unknown and invisible, and therefore difficult to study, particularly dark matter. Since dark matter is undetectable via traditional means such as telescopes, we have to consider alternative methods to indirectly study the fundamental nature of dark matter. The main way scientists observe dark matter via its gravitational influence. Since we can see these gravitational effects, it follows that there has to be other gravitating objects that do not emit or reflect light. Black holes are one such object. They are a region of spacetime with such strong gravitational fields that neither electromagnetic radiation (light) nor particles can escape. In this work, we go into more detail on how to detect dark matter and consider the possibility of primordial black holes as candidates for dark matter by systematically looking at constraints developed through various observations.

Dark Matter

Before we can explore what candidates of dark matter are, we must first understand how dark matter can be detected and what the implications this empirical evidence means. In this section we outline some important features of dark matter and how black holes could account for its effects.

Observing Gravitational Influences of Dark Matter

One method of observing dark matter is by looking at its gravitational influence on visible matter such as stars' orbital speeds with near circular orbits in spiral galaxies.

If a star in circular orbit around a galaxy has a radius of orbit R and orbital speed v, the star experiences an acceleration of

$$a = \frac{v^2}{R} \tag{1}$$

towards the center of the galaxy. If the acceleration is provided by gravitational attraction of the galaxy, then

$$a = \frac{GM(R)}{R^2} \tag{2}$$

assuming the mass distribution of the galaxy is spherically symmetrical. $M(R)$ is the mass contained within a sphere of radius R centered around a galactic center, and G is the gravitational constant which is $G \approx 6.67 \times 10^{-11} Nm^2 kg^{-2}$. From the two previous equations, we get

$$v = \sqrt{\frac{GM(R)}{R}} \tag{3}$$

If stars contributed all or most of the mass in a galaxy, the orbital speed v of a star in circular orbit around the center of its galaxy would fall as $v \propto 1/\sqrt{R}$ at a large orbital

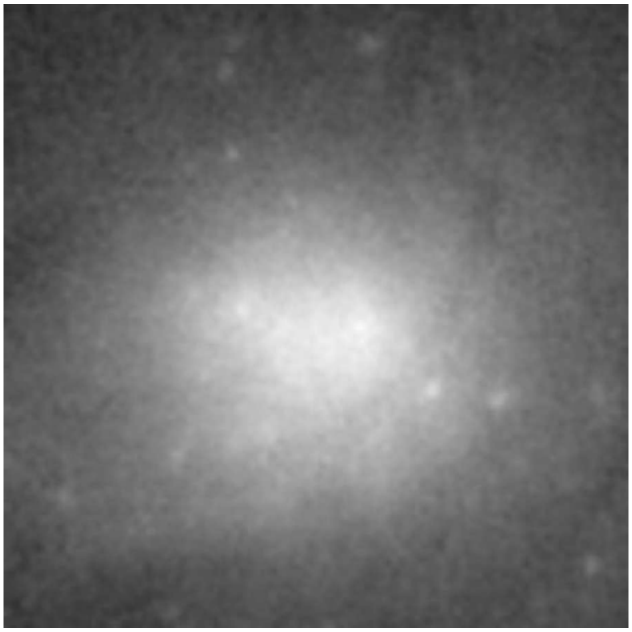

Fig. 1 The Coma cluster in x-ray light. The image shown is 35 arcminutes across, which is equal to $\sim 1Mpc$ at the distance of the Coma cluster.
From the ROSAT x-ray observatory. This figure was produced by Raymond White, using NASA's SkyView facility[1].

radius beyond the radius of the galaxy[2]. However, the orbital speed of stars within a spiral galaxy can also be determined through observations. The orbital speed $v(R)$ out to a radius $R = 24$ $kiloparsec(kpc)$ can be determined by observing emission lines from regions of hot (high-energy) ionized gas in M31 (Andromeda Galaxy), where $1 parsec \approx 3.26\ light - years \approx 3.08 \times 10^{16} m$. However, visible light beyond $R = 24\ kpc$ was too faint to measure the redshift—akin to the doppler shift in sound frequencies, the redshift is an increase in wavelength due to the universe in expanding[3].

Measuring the relative velocity of astronomical objects lets us understand their dynamic and therefore the gravitational influence of the dark matter surrounding it. Within galaxies, the magnetic dipole moment or "spin" of the protons and electrons in a hydrogen atom can be aligned, or misaligned. When the proton and electron are aligned, they "spin" in the same direction in a higher energy state than when they are anti-aligned and spinning in opposite directions. When the protons and electrons transition from aligned to anti-aligned, there is a small difference in energy between the two states and this shift releases an abundance of photons with a quantized wavelength of $21cm$; however, since this occurred in the earlier universe, all of it has been redshifted beyond $21cm$. By measuring the redshift, it is possible to determine how fast objects are moving away from each other. It was later found that orbital speed stayed at a nearly constant value of $v(r) \approx 230\ kms^{-1}$ out to $R \approx 30kpc$ by detecting the small amount of atomic hydrogen at $R > 24kpc$ through its emission line at $\lambda = 21cm$[4]. Since the stars and hot ionized gases at large radii in M31 galaxy have an orbital speed that is greater than if only visible matter were present. It can be reasonably deduced that there is halo of dark matter surrounding the galaxy, otherwise known as a dark halo, and this can be applied to all parts of the universe.

When stars and gases are insufficient to hold a galaxy cluster together due to gravitational pull, dark matter's gravitational impacts can be observed. Scientists first observed the Coma cluster of galaxies in the 1930s and found that its visible stars and gas did not generate enough gravitational attraction to hold the cluster together. As a result, the cluster has a large amount of dark matter to keep its galaxies from breaking apart[5].

Using the virial theorem—which computes the average kinetic energy of a system with an given potential energy—to estimate the mass of the Coma galaxies, with many assumptions due to partial information on the cluster, the Coma cluster's mass can be estimated to be approximately $2 \times 10^{15} M_{\odot}$ for 1 $M_{\odot} = 1.988 \times 10^{30} kg$ or one solar mass. Following this, less than 2 percent of the cluster's mass consists of stars ($M_{Coma,Star} = 3 \times 10^{13} M_{\odot}$) and only ten percent consists of hot intracluster gas, which accounts for most baryonic material in galaxy clusters ($M_{Coma,Gas} = 2 \times 10^{14} M_{\odot}$). The luminosity of the Coma cluster, which is $L = 8 \times 10^{12} L_{\odot}$ where $L_{\odot} \approx 3.846 \times 10^{26}\ w$ is the solar luminosity, refers to the average electromagnetic energy emitted by the Coma cluster. The mass-to-light ratio of star is the quotient between the star's mass and luminosity. The Coma cluster has a mass-to-light ratio of $\approx 250 M_{\odot}/L_{\odot}$, which is anomalously greater than the mass-to-light ratio of our galaxy. Since there's a greater concentration of mass in the coma cluster, these values indicate a vast reservoir of dark matter. This is further confirmed by the hot, x-ray emitting intracluster gas shown in Figure 1 staying in place; if there was no dark matter to gravitationally anchor the gas, the gas would have expanded beyond the cluster. Applying the virial theorem to other galaxies results in a mass-to-light ratio range of $200 M_{\odot}/L_{\odot} \rightarrow 300 M_{\odot}/L_{\odot}$. This data indicates that the Coma cluster contains the expected amount of dark matter[1].

Gravitational Lensing of Dark Matter

Einstein's general theory of relativity posits that an object's gravity could affect not only the trajectory of matter, but also the trajectory of photons. Dark matter can therefore operate as a gravitational lens to focus and bend light. For example, consider a dark halo surrounding a galaxy which consists of clusters of high matter density which we call massive compact halo objects (MACHOs). If a photon passes by such an object,

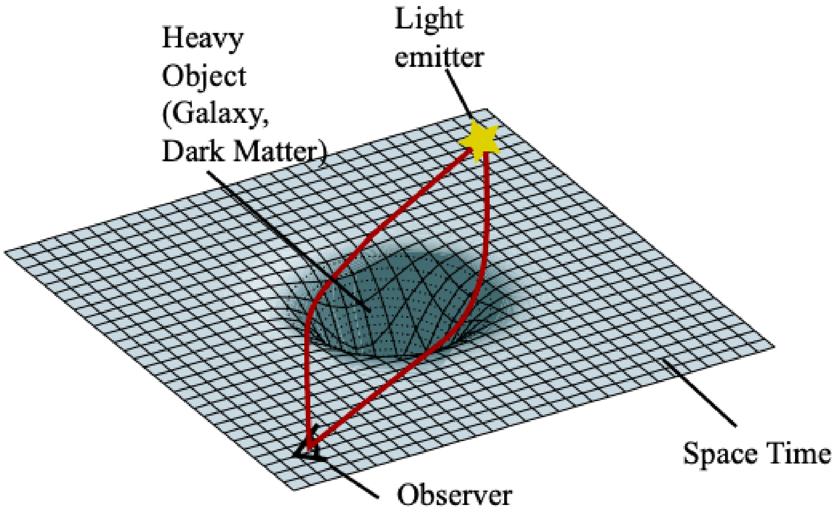

Fig. 2 Diagram of the gravitational lensing effect[6].

the local curvature of space time will cause the photon to be deflected at an angle of $\alpha = \frac{4GM}{c^2 b}$ where M is the mass of the compact object, b is the impact parameter, and c is the speed of light in vacuum.

The image formed is a perfect ring with an angular radius of

$$\theta_E = \left(\frac{4GM}{c^2 d}\frac{1-x}{x}\right)^{\frac{1}{2}} 7 \tag{4}$$

if the MACHO is precisely along the line of sight between the observer and the lensed star where M is the mass of the lensing MACHO, d is the distance from the observer to the lensed star, and $xd(where 0 < x < 1)$ is the distance from the observer to the lensing MACHO. The angle θ_E is known as the Einstein radius. However, given the small size of the radius of the Einstein radius, gravitational lensing is not effective. Based on data from the Large Magellanic Cloud, it has been determined that the chance of any given star being lensed at any given time, even if the dark halo of our galaxy were fully made up of MACHOs, would still only be $P \sim 10^{-7}$, or one in ten million[7].

Primordial Black Holes as Dark Matter

Black holes exist over a wide range of mass scales. There is overwhelming evidence for intermediate-mass black holes and supermassive black holes. However, these can only provide a small fraction of the dark matter density. Primordial black holes can possibility provide significantly more of the dark matter density, which might provide an explanation to the origin of dark matter.

Black Holes

Black holes are regions of spacetime warped around concentrated masses whose gravitational fields are strong enough to prevent even light from escaping. The concept of an object capturing light and thereby becoming dark can be traced back to studies by John Michell and Pierre-Simon La-place in the 18th century. By calculating the escape velocity of a light particle using Newtown's gravitational laws, they predicted the possibility of stars so dense that light could not escape: a "dark star." However, at the time, light was believed to consist of corpuscles, a minute particle historically regarded as the basic constituent of light. The discovery of light's form as a wave made it unclear how the Newtonian gravitational field affected light, undermining the concept of dark stars. In late 1915, Albert Einstein proposed the general theory of relativity, providing an understanding of how light in the form of a wave would behave under the influence of a gravitational field[9–11]. Due to significant technological advances and new observational facilities such as the LIGO experiment which, for the first time in 2015, detected gravitational waves from merging binary black holes, the concept of black holes is now widely

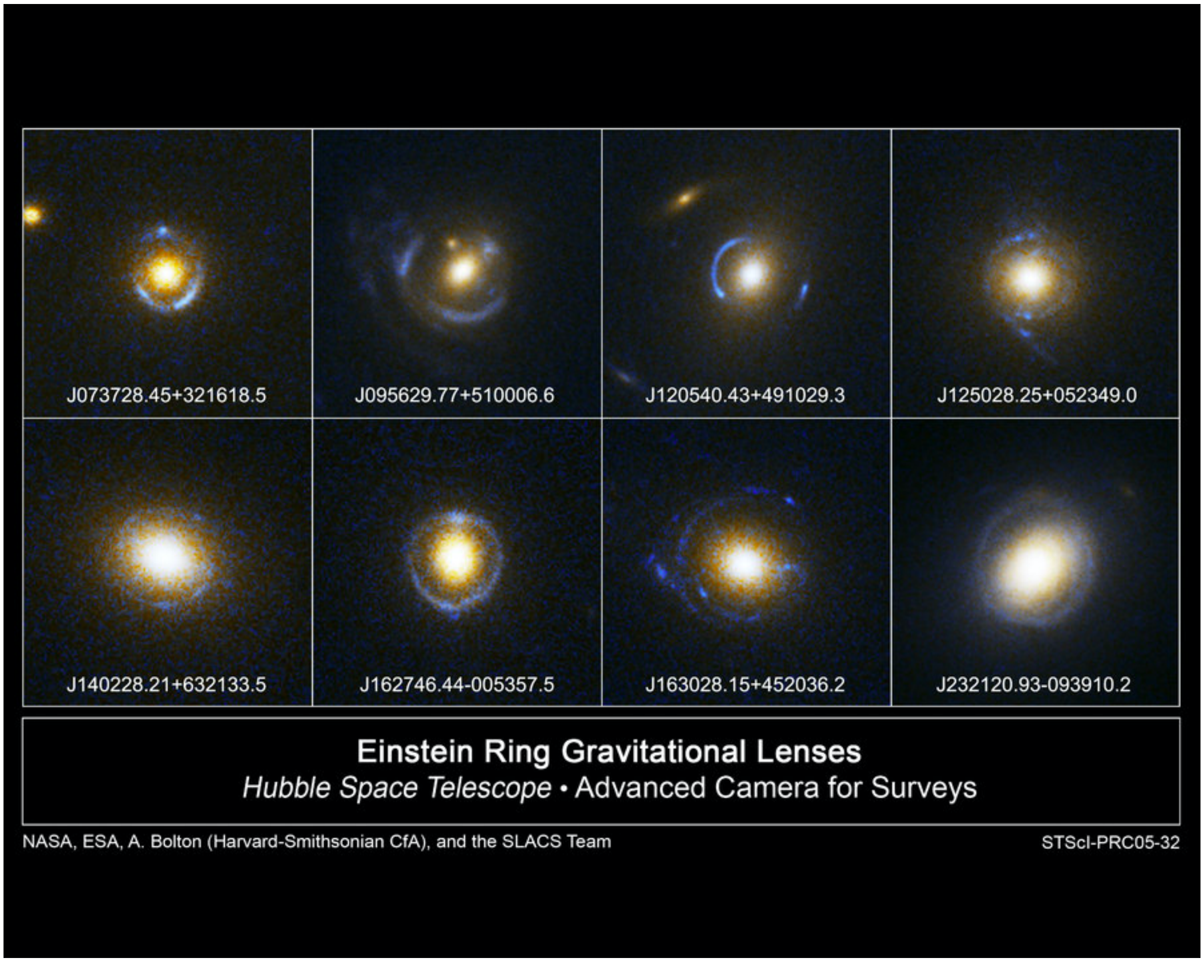

Fig. 3 The rings here are more distant galaxies lensed by foreground galaxies[8].

accepted in astronomy[12].

For a region of mass M to collapse under its own weight and form a black hole, the radius of the massive region needs to fall within its Schwarzschild radius of

$$R_S = \frac{2GM}{c^2} \tag{5}$$

where c is the speed of light in a vacuum and G is the gravitational constant. R_S sets the boundaries for the event horizon of a black hole, a point past where light can no longer escape the gravitational force of the black hole. For a region of mass M to collapse within this radius, it would require a density of

$$\rho \sim \frac{c^6}{G^3 M^2} \tag{6}$$

Primordial Black Holes

Black holes forming in the early universe are termed as "primordial." Primordial black holes (PBHs) most likely formed from primordial density fluctuations during the inflationary period early universe $(10^{-36}s - 10^{-33s})$ as clumps of matter grew to cosmological sizes as they attracted particles[13].

On large scales, which refers to scale of $100Mpc$ or more, the universe is isotropic and homogenous. Isotropic means that there no preferred directions in the universe: no matter the direction, the universe looks the same. Similarly, homogenous means that there are no preferred locations in the universe: no matter the location, the universe looks the same. The standard Big Bang theory is based on this idea. However, the Big Bang theory leaves number of puzzles, namely the "flatness problem" and "horizon problem." To resolve these issues, physicists pioneered the idea of cosmic inflation, which explains why the universe is flat and why the CMB radiation is uniform. Additionally, the inflation theory explains the origin of structures in the universe—the result of magnified quantum fluctuations, or a temporary change in the amount of energy in a point of space[14].

These overdense regions would eventually stop expanding after entering the particle horizon before contracting under their own weight. Additionally, reheating, which is the production of standard model matter following a period of accelerated expansion, which may also produce PBHs[15].

Since the cosmological density at time t after the Big Bang—a theory that marks the origin of the universe from the expansion of matter from a state of high density—is

$$\rho \sim \frac{1}{Gt^2} \tag{7}$$

PBHs would initially have around the cosmological horizon mass:

$$M \sim \frac{c^3 t}{G} \sim 10^{15}\left(\frac{t}{10^{-23} s}\right) g\ ^{15} \tag{8}$$

By this definition, PBHs can span an enormous mass range. Due to their miniscule size in the early universe, through Hawking radiation black holes radiate thermally with a temperature

$$T = \frac{hc^3}{8\pi GMk} \approx 10^{-7}\left(\frac{M}{M_\odot}\right)^{-1} K\ ^{16} \tag{9}$$

$$\tau(M) \approx \frac{hc^4}{G^2M^3} \approx 10^{64}\left(\frac{M}{M_\odot}\right)^{3} yr\ ^{16} \tag{10}$$

Therefore, PBHs initially lighter than $M_* \sim 10^{15}\ g$ formed before $10^{-23}\ s$ and were the size of a proton, would have evaporated by now. Because of this, attention in recent years has focused more on PBHs larger than $10^{15}\ g$. These are unaffected by Hawking radiation mainly due to the possibility that PBHs could provide dark matter which makes up 25% of the critical density (the average density of matter required for the universe to stop expanding). Black holes which formed at late times cannot provide all the dark matter because they're formed from baryons and the big bang nucleosynthesis constraint states that baryons are at most 5% of the critical density[17].

Constraints on Primordial Black Holes

The first constraint considered is that from evaporation, and their observational signature. PBHs of mass M that not yet evaporated completely have constraints on the fraction of dark matter $(f(M))$ that they could make up. It can be derived from partial evaporations, gravitational-lensing experiments, dynamical effects, and accretion, under the assumption that PBHs cluster in galactic halos in the same way as cold dark matter, unless there is less than one per galaxy. However, all the constraints all have degrees of uncertainty[18].

Evaporation Constraints

PBHs of initial mass M will evaporate through the emission of Hawking radiation on a timescale $\tau \propto M^3$[19]. When M is below $M_* \approx 5 \times 10^{14}\ g$, they cannot be dark matter as it would have evaporated by now as the evaporation time is less than the present age of the universe. PBHs in the range of $M_* < M < 1.005M_*$ have not evaporated yet, but their current mass is below the mass of $0.4M_*$ at which quarks and gluons are emitted. This is relevant because this mass is the threshold at which quarks and gluons are emitted during Hawking radiation.

For a PBH of mass M evaporating, the instantaneous spectrum for primary (non-jet) photons is

$$\frac{dN^P_\gamma}{dE}(M,E) \propto \frac{E^2\sigma(M,E)}{e^{EM}-1} \propto \begin{cases} E^3M3 & (E < M^{-1}) \\ E^2M2e^{-EM} & (E > M^{-1}) \end{cases}\ ^{20} \tag{11}$$

where $\sigma(M,E)$ is the absorption cross-section for photons of energy E with units $h = c = G = 1$, which gives an intensity of

$$I(E) \propto F(M) \times \begin{cases} E^4M2 & (E < M^{-1}) \\ E^3Me^{-EM} & (E > M^{-1}) \end{cases}\ ^{20} \tag{12}$$

This energy peaks at $E_{max} \propto 1/M$ with a value of $I_{max}(M) \propto f(M)M^{-2}$. The observed intensity is $I_{obs} \propto E^{-(1+\varepsilon)}$ with ε between 0.1 and 0.4. If PBHs are radiating light, their intensity needs to be below the observed threshold. Putting $I_{max}(M) < I_{obs}[E_{max}(M)]$ gives

$$f(M) < 2 \times 10^{-8}\left(\frac{M}{M_*}\right)^{3+\varepsilon} (M > M_*)\ ^{20} \tag{13}$$

Using positron data from the Voyager 1, it is possible to constrain evaporating PBHs of $M < 10^{16}\ g$ to obtain the bound of $f(M) < 0.0012$[22]. The observation of the Galactic Center $511 KeV$ γ-ray line can derive stringent constraints for PBHs that show PBHs contribute less than 1% of the dark matter density[23].

Lensing Constraints

Primordial black holes can also be constrained through the constant monitoring of stars since heavy black holes can change stars' trajectories. If PBHs make up a significant fraction of dark matter, it is expected that there are many microlensing events. However, since there aren't many, this leads

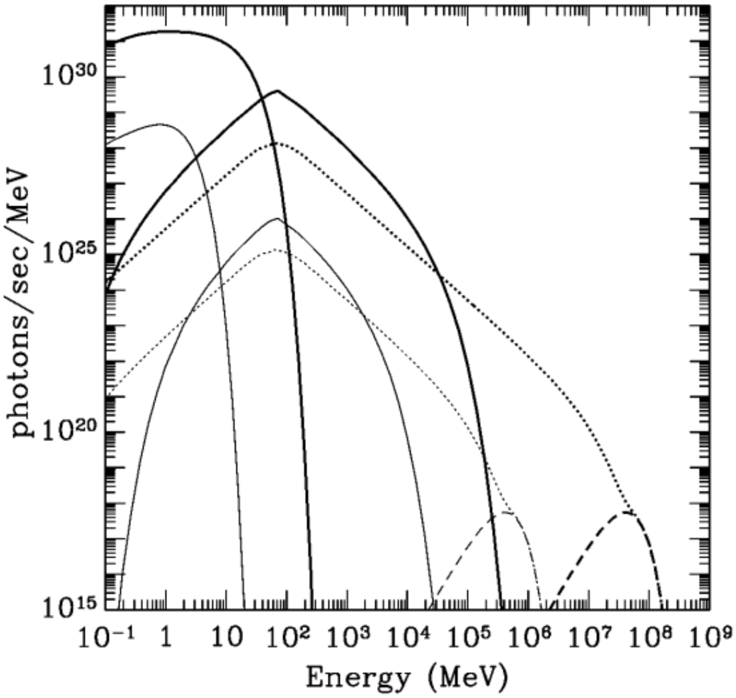

Fig. 4 "Instantaneous emergent spectra. Thick lines are for a $M = 10^9\ g$ black hole. Thin lines for $M = 10^{11}\ g$. The solid lines are spectra which include photosphere: the ones peaking at about $100MeV$ are the emergent spectra of QCD photosphere, the ones peaking at about $1MeV$are for the QED photosphere. The dotted lines are the direct fragmentation results of MacGibbon and Webber, and the dashed lines are the direct photon emission spectra. The actual full spectrum is the addition of the two solid lines."[21].

to various constraints on the fraction of dark matter that black holes and their mass could account for.

Kepler data from the observation of Galactic sources imply a limit in the planetary mass range $f(M) < 0.3$ for $2 \times 10^{-9} M_{odot} < M < 10^{-7} M_{odot}$ while observations of M31 via the Subaru Hyper Suprime-Can obtain a stricter bound of $10^{-10} M_{odot} < M < 10^{-6} M_{odot}$[24,25]. Cosmologists have used the lack of lensing in type Ia supernovae (a supernova that occurs in a binary system where there is a white dwarf) to constrain any PBH population, deriving a bound of $f(M) < 0.35$ for $10^{-2} M_{odot} < M < 10^4 M_{odot}$ using current light-curve data[26]. However, limit can be weakened if PBHs have an extended mass function—which considers the matter surrounding a black hole, thus dispersing the gravity and weakening the bounds—or are clustered[27].

Dynamical Constraints

PBHs can also disrupt the dynamic of other systems. This interaction produces constraints for PBHs through the observation of regions with known dark matter densities and looking at the rate at which these systems are disrupted.

One such example is constraining PBHs by considering their capture by white dwarfs or neutron stars at the center of globular clusters (tightly gravitationally bound group of stars). In this situation, it has been argued that this excludes PBHs with mass in range $10^{14} - 10^{17}\ g$ from providing dark matter [this lower mass range is because we suspect there shouldn't be much dark matter in globular clusters]; however, the dark matter density in globular clusters is now known to be much lower than assumed in the analysis[28,29]. Meanwhile, it has been argued that a PBHs transiting through a white dwarf causes the white dwarf to explode as a supernova; therefore, $10^{19} - 10^{20}\ g$ PBHs cannot provide dark matter[30]. But hydrodynamical simulations, a method that simulates interactions between structures by treating large objects as deformable bodies suggest otherwise[31].

At higher mass scales, there are a variety of dynamical constraints—many involving the destruction of astronomical objects by a PBH passing nearby. If PBHs have density p and velocity dispersion v (spread of values for multiple astronomical objects), while the objects have massM_c, radiusR_c, velocity dispersion v_c, and survival timet_L, then the constraint has the form:

$$f(M) < \begin{cases} \frac{M_c v}{G M_p t_L R_c} & \left[M < M_c(\frac{v}{v_c})\right] \\ \frac{M_c v}{p v_c t_L R_c^2} & \left[M_c(\frac{v}{v_c}) < M < M_c(\frac{v}{v_c})^3\right] \\ \frac{M_c v_c^2}{p R_c^2 v^3 t_L} exp\left[(\frac{M}{M_c})(\frac{v_c}{V})^3\right] & \left[M > M_c(\frac{v}{v_c})^3\right] \end{cases} \quad 32 \tag{14}$$

Note that this equation is robust for many kinds of physical systems, such as binary systems or globular clusters, and the limits on dark matter fraction will change the parameters M_c, R_c, and v_c. The three limits correspond to disruption by multiple encounters (from many smaller black holes), one-off encounters (experience disruption once from a medium-sized black hole), and non-impulsive encounters (from massive black holes that don't impart all their energy due to their speed), respectively given that there is at least one PBH in the relevant environment[32].

For example, this argument can be applied to the survival of globular clusters against tidal disruption by passing PBHs, a process where an object, which is sufficiently close to a black hole, experiences strong gravitational force. The survival of globular clusters gives a limit $f(M) < (\frac{M}{3} \times 10^4 M_\odot)^{-1}$ for $M < 10^6 M_\odot$, though it depends sensitively on the cluster's mass and radius[33]. For giant cluster PBHs, too large to reside in galactic halos, the survival of galaxies in clusters against tidal disruption provides a limit $f(M) < (\frac{M}{7} \times 10^9 M_\odot)^{-1}$ for $M < 10^{11} M_\odot$[34].

Gravitational-Wave Constraints

A population of massive PBHs would be expected to generate gravitation-wave background (GWB)—a gravitational-wave signal—which is interesting, especially if, at the present epoch, a population of binary black holes combines due to gravitational-radiation losses[35]. Generated by accelerated masses, gravitational waves are disturbances in spacetime which radiate outwards as waves from their source. As the crests and troughs of a gravitational wave pass through space, there are distortions in spacetime making particles oscillate cruciformly. Conversely, the non-observation of GWB provides constraints on the fraction of dark matter in PBH.

The rate of gravitational-wave events observed by LIGO/Virgo have been claimed by to match the expected merger rate of PBHs providing dark matter[36]. But, unless PBHs provide a small fraction of dark matter the merger rate is tension with the cosmic microwave background distortion[37]. However, these two arguments assume that binaries formed in the early universe whereas the second assumes that they form after the formation of galaxies. However, if binaries formed in the early universe and survived until the present, it can be inferred that the binary merger rate from gravitational capture in present-day halos should be subdominant if binaries formed in the early universe and survived until the present[38]. If this is the case, the merger rate is less than the current LIGO/Virgo upper limit if and only if $f(M) < 0.01$ for $10 - 300M_{odot}$ PBHs[39]. But it has been claimed that early binaries merge well before LIGO/Virgo observations, weakening the limit or removing it altogether[40].

Conclusion

As far back as the 1930s, astronomers proposed the idea that galaxies and clusters of galaxies had dark matter, a stabilizing force that held these structures together gravitationally. In the most recent decade, researchers have opened to the possibility of dark matter consisting of primordial black holes which emerged from the Big Bang.

Most recent work derives from the steadily increasing number of gravitational-wave detections coming from LIGO/Virgo. Additionally, we review the correlations between primordial black holes as fraction of dark matter based on existing observations. The next couple decades could bring observational evidence that either confirms the possibility of PBHs as dark matter or eliminates it. Incoming data from the James Webb Space Telescope (JWST) will also help improve our understanding on the nature of dark matter. PBHs may also contribute a fraction of dark matter, but it is hard to rule out everything.

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