Application of holography to the black hole information problem

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Hawking radiation and the thermal radiation of possibly mixed states is at odds with the principle that pure states must only evolve into pure states – unitarity. Unitarity functions as a fundamental principle in quantum mechanics, but the contradictions evoked by Hawking radiation assert that unitarity may not be so universal. The conjectured relationship between 5D Anti-deSitter Space time (AdS) and 4D conformal field theory (CFT) postulated by AdS/CFT allows one to use its duality to resolve the information problem. This review article will explore this method. The findings suggest that unitarity is conserved and remains a fundamental principle in quantum field theory. This is significant as it provides a better understanding of what a theory of quantum gravity would look like.

Introduction

The Schrödinger equation describes the evolution of a quantum mechanical system over time. This evolution over time is deterministic – that is, given a set of conditions, one can trace the past and future states of the wave function with the unitary operator. This determinism is the basis of unitarity as a principle in quantum field theory and supports a more general conservation of quantum information. Unitarity contends that for an initial state $|\Psi(t)\rangle$ from a time t is given by the following,

$$|\Psi(t+\Delta t)\rangle = U(\Delta t)\,|\Psi(t)\rangle \quad (1)$$

A wave function describes the probable quantum states of a system. In the case of pure quantum states, there is a definite wave function that can describe the system. However, for mixed quantum states, there is no definite wave function that can describe the quantum system. Instead, the system can be defined by a number of probable wave functions[1]. Specifically, pure quantum states will and should only evolve into pure quantum states through the Schrödinger equation. However, the validity of unitarity as an indispensable principle is called into question through the thermodynamic behavior of black holes[1].

The classical interpretation of black holes allows one to develop black hole thermodynamic laws, like the laws of thermodynamics. The zeroth law of thermodynamics contends that temperature for bodies at thermal equilibrium is a constant value. Similarly, the surface gravity (κ) of black holes is constant across the event horizon of the black hole[2]. Therefore, this constitutes the zeroth law of black hole thermodynamics. The surface gravity is expressed – in SI units - as

$$\kappa = \frac{c^4}{4GM} \quad (2)$$

where κ is the surface gravity, c is the speed of light, G is the universal gravitational constant, and M is the mass of the black hole[3]. Statistical mechanics also specifies that the total entropy of a closed system can never decrease. The classical interpretation of black holes postulates that the black hole's event horizon surface area (A) can never decrease and can only increase with infalling matter[2,4]. So, based on this classical analogy, it is reasonable to relate the entropy of a black hole as a function of the area of its event horizon. The area of a Schwarzschild black hole's event horizon (A) can be expressed by the following formula:

$$A = 4\pi r_s^2 = \frac{16\pi G^2 M^2}{c^4} \quad (3)$$

where the Schwarzschild radius $r_s = 2GMc^{-2}$

Differentiating the area with respect to mass (M) and putting (equation 2) in (equation 3)

$$dM = \frac{\kappa}{8\pi G} dA \quad (4)$$

Meanwhile, the second law of classical thermodynamics is given as,

$$dE = T\,dS \quad (5)$$

where T is temperature, E is energy, and S is the entropy of the system. Comparing this to (equation 4), one notices a similarity. For classical stationary Schwarzschild black holes, the surface gravity (κ) is constant over the event horizon just as temperature is constant for a thermodynamic system in equilibrium, and the surface area of the black hole (A) is positive

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and never decreases just as entropy for a thermodynamic system. Hence, the factor $\frac{\kappa}{8\pi}$ is comparable to T and dA is comparable to dS[3].

While these derivations are valid for a classical understanding of black holes, they become more nuanced when considering quantum effects, particularly Hawking radiation. Hawking radiation is the eventual thermal radiation from black holes. Quantum vacuum fluctuations caused by the strong gravitational field in a black hole lead to the spontaneous production of a pair of particles, one of which falls into the black hole, while the other is radiated out as Hawking radiation[5]. The temperature of this Hawking radiation was calculated and is expressed as follows[6],

$$k_b T_H = \frac{\hbar\kappa}{2\pi c} \tag{6}$$

From (equation 4),

$$dM = \frac{\kappa}{8\pi G}dA = \frac{\hbar\kappa}{2\pi c}\frac{c}{4\hbar G}dA$$

$$dMc^2 = k_b T_H \frac{c^3}{4\hbar G}dA$$

Equating this to (equation 5),

$$k_b T_H \frac{c^3}{4\hbar G}dA = T_H dS$$

$$S = \frac{Ak_b c^3}{4\hbar G} \tag{7}$$

This cements black hole thermodynamics by making black holes more consistent with statistical mechanics, but the concept of hawking radiation and subsequent black hole evaporation is more problematic. Since the contents and infalling matter in a black hole are hidden from an external observer and the only measurable features of a black hole – by the "no-hair theorem" – are its mass, angular momentum, and electric charge, hawking radiation implies that some information about the fallen matter disappears within the black hole and is lost through radiation[7]. This is because the thermal radiation bath is independent of the initial state of the black hole and is therefore a mixed state. Additionally, this implies that pure quantum states would evolve into mixed quantum states through hawking radiation, which calls unitarity into question because pure quantum states should only evolve into pure quantum states[1]. With mixed quantum states – the possibility of multiple probable quantum states as opposed to one – the situation is no longer deterministic, and the loss of information of states and the violation of unitarity are inevitable[1,3]. This is known as the black hole information problem. The problem not only concerns whether black holes uphold unitarity, but also whether unitarity itself is a universal quantum principle. The black hole information problem says otherwise, which is why the problem is important for a complete understanding of quantum field theory.

There are multiple proposed solutions to the black hole information problem. Some are listed below:

- Holography argues that information is not lost. It uses the duality of 5D Anti-deSitter Spacetime and 4D SYM gauge theory to show that black hole evolution is unitary. This theory will be the focus of this paper[1,8].
- A Planck-sized remnant is the remainder of the black hole after it has completely radiated. Only the "lost" information exists in the remnant of the black hole[1,9].
- Unitarity is no longer valid in the context of black holes. Some argue that unitarity – and the conservation of information – are not pivotal laws of quantum mechanics[10].

This paper will review the application of the holographic principle to the information problem.

Methods

This paper, as a review, relies primarily on targeted literature review methodologies within the overlapping fields of holography and black hole information studies. The primary database used was arXiv. Supplementary books such as Natsuume's AdS/CFT User Guide and lecture notes were used for an overview of the developments in holography. Resources were selected based on the number of citations and recency in the respective field. In the first stage, papers on the development of black hole thermodynamics and particle creation in black holes were perused. Ideas from these papers feature predominantly in the introduction. The next stage of research focused on unifying AdS and CFT through string theory, and the papers used included Juan Maldacena's "Large N limit of superconformal field theories and supergravity" and the Polchinksi lecture notes on the black hole information problem. Lastly, the application of these ideas to the information problem are discussed in the aforementioned lecture notes and[9].

Results

AdS/CFT Correspondence

AdS/CFT was first proposed by Juan Martín Maldacena as a means of describing the connection between Anti-deSitter spacetime and a conformal field theory[8]. This connection is realized with the help of a dictionary that maps quantities in anti-deSitter spacetime to those in a conformal field theory. In a special case of AdS/CFT, which will be studied in this section, five-dimensional anti-deSitter spacetime relates to four-dimensional N=4 supersymmetric Yang-Mills theory[3,8]. The complications with black hole radiation rise from the difficulty to reconcile relativistic effects with particle interactions within the black hole. Hence, the AdS/CFT correspondence allows one to work with AdS black holes in the language of particle interactions in conformal field theory, acting as a reconciliation to an extent. This relation through the duality of AdS/CFT permits the usage of the holographic principle to resolve the black hole information problem. In the ensuing sub-sections, the tools to develop the correspondence (duality) will be discussed and will be used in the solution section.

Anti-deSitter Spacetime

The cosmological constant is a coefficient that appears in Einstein's relativistic field equations to explain the expansion of the universe. It describes vacuum energy density in space and is determined to be responsible for the accelerating expansion of the universe. It is denoted by Λ[11]. AdS spacetime is a maximally symmetric solution to Einstein's Field Equations with a negative cosmological constant. A maximally symmetric solution is one which is isotropic – physically the same in all directions at a point – and homogeneous – physically the same at all points in space. The negative cosmological constant ($\Lambda < 0$) allows for constant negative scalar curvature – that is, the curvature of the manifold is of the same negative value at all points on the spacetime[12].

In addition to gravity, there is an attractive force between objects in Anti deSitter spacetime due to the negative cosmological constant in contrast to the repulsive force opposing gravity in deSitter spacetime. Consequently, it possesses hyperbolic geometry[3]. A maximally symmetric spacetime is one which possesses both homogeneity and isometry. Homogeneity means that the laws of physics are symmetric under spatial and time transformations, and isotropy means that the laws of physics are symmetric under rotation – that is, they are rotationally invariant. A d-dimensional AdS spacetime is denoted by AdS_d.

A metric allows one to find distances or angles between two points on a given manifold. Specifically, the Poincare metric allows us to describe surfaces with constant negative curvature. Therefore, AdS5 can be described by the following Poincare metric[3]:

$$\frac{ds_5^2}{L^2} = r^2(-dt^2 + \vec{x}^2) + \frac{dr^2}{r^2} \tag{8}$$

Where L is the AdS radius, t represents the time coordinate, and $\vec{x}$ represents the spatial three-dimensional vector. This metric is of importance because one can see the 4-dimensional flat metric $(-dt^2 + \vec{x}^2)$ in the Poincare metric. We can define a gauge theory with this metric, an idea which will be explored in the solution section. As $r \to \infty$, one approaches the boundary of AdS. It is important to note that one can consider black holes in AdS, an idea which will become important in the solution section. With the development of these tools, one now has a framework on which the evolution of black holes can be studied.

Conformal Field Theory

A quantum field theory is a description of particles that combines quantum mechanics and relativity. It treats particles as excitations of quantum fields that pervade space[13]. All quantum field theories obey some underlying symmetry. Examples of quantum field theories include Quantum Electro-Dynamics (QED) and Quantum Chromodynamics (QCD). Another example is a conformal field theory. A conformal field theory is one which remains invariant under conformal transformations, which are transformations that preserve angles but not necessarily lengths[14]. An example of such a theory is N=4 super Yang-Mills Theory which turns out to be related to AdS_5 through the duality[14,15].

To relate curved spacetime and conformal field theories, it is essential to impose scale invariance, invariance of gauge theories under changes in energy scales, on both[14]. Gauge Theory, closely related to QFTs, explains fundamental interactions among elementary particles. The general form of a metric for a curved five-dimensional spacetime related to a four-dimensional gauge theory is as follows[3],

$$ds^2 = \Omega(w)^2(-dt^2 + \vec{x}^2) + dw^2 \tag{9}$$

where $\Omega(w)$ represents a factor to determine the type of curved spacetime that corresponds to the four-dimensional gauge theory. Given that the metric of 4-dimensional space $(-dt^2 + \vec{x}^2)$ is contained in the metric of 5-dimensional AdS spacetime[16], this metric possesses Poincare invariance. It simply means that the metric possesses translational symmetry, rotational symmetry, and symmetry under Lorentz transformations[17]. On imposing scale invariance, the metric becomes that of Anti-deSitter spacetime. Therefore, the resulting metric possesses both Poincare invariance and scale invariance, which generalizes to become conformal invariance under the group SO (2,4) – a group of transformations under which the metric is conformally invariant[3].

There is an underlying connection between the two theories, and this is elucidated through the relation of parameters via string theory. N_c is the number of colors in a symmetry group. It essentially describes how large the symmetry of a group is. At the large N_c limit of gauge theory, strong interactions can be described using string theory, but string theory is not quantizable (describable) in four dimensions[15]. Nonetheless, considering string theory for higher dimensions ($d \geq 5$) allows us to describe large N_c strong interactions in AdS_5. Thus, string theory forms the dictionary to relate terms in both concepts in the following ways[18]:

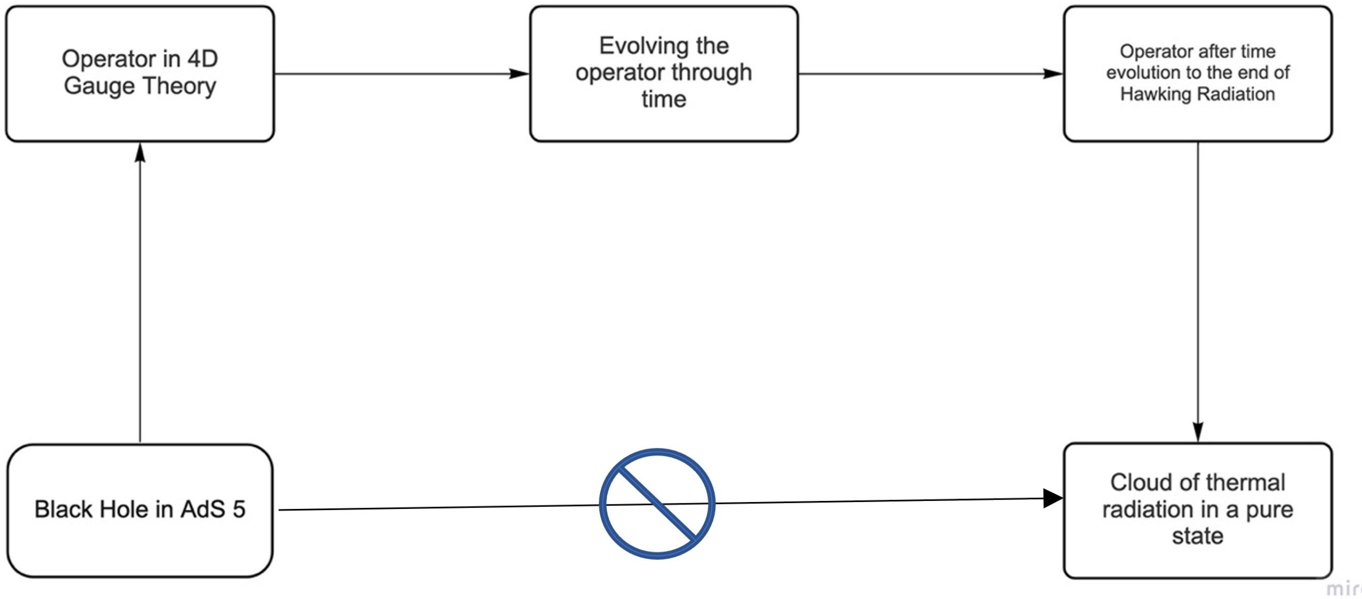

Fig. 1 A summary of the procedure to resolve the problem

$$N_c = \frac{\pi}{2}\frac{L^3}{G_5} \quad (10)$$

$$N_c = \frac{\pi}{2}\frac{L^3}{G_5}\lambda = \left(\frac{L}{l_s}\right)^4 \quad (11)$$

where N_c is the number of colors, L is the AdS radius, G_5 is the gravitational constant in 5 dimensions, λ is the t'Hooft coupling constant in non-Abelian gauge theories, g_s is the string coupling constant, and l_s is the length of the string. The main claim of AdS/CFT is that the conformal gauge theory "lives" on the boundary of the 5-dimensional AdS spacetime. The identification of a five-dimensional theory and a four-dimensional theory living on its boundary is known as holography[8]. The equations above provide us with a dictionary that relates objects in the gauge theory to those in AdS. Essentially, we have considered a four-dimensional conformal field theory and a five-dimensional spacetime which can be related by parameters developed via string theory. Therefore, the complications due to gravitational interactions in the information problem, which could not be solved due to our incomplete understanding of quantum gravity, can now be solved by this correspondence.

Discussion

We want to know whether black holes maintain unitarity through Hawking radiation. However, from a gravitational perspective, one cannot tell if black holes evolve unitarily. Because no theory of quantum gravity exists and because quantum mechanical effects and gravitational effects cannot be ignored with regards to black holes, the question of whether unitarity is upheld remains. This is where the duality of AdS/CFT and our dictionary comes in.

Consider a black hole existing within AdS spacetime. Because 5-dimensional anti-deSitter spacetime has an equivalent description to four-dimensional gauge theory via the duality[8], one can map the AdS black hole to an object – more specifically, an operator - in four-dimensional gauge theory using the AdS/CFT dictionary.

Four-dimensional gauge theories maintain unitarity, so one can evolve the operator unitarily through enough time such that the black hole in AdS spacetime has evaporated completely through Hawking radiation[19]. Then, once radiation is complete and the black hole has radiated out its contents into a thermal bath of radiation, the operator can be again mapped back to the AdS Black Hole[19]. The following chart (Figure 1). offers a general idea of the process.

An important consequence is that such a computation has shown us that the system evolves into a cloud of thermal radiation in its pure state. This implies that pure states that enter a black hole are radiated out as pure states, thereby upholding the principle of unitarity in Section 1[19]. This is a significant result because this shows that unitarity is a fundamental principle in quantum field theory – that it is not violated by black holes - and offers a better understanding of what a quantum gravity theory would look like.

As unitarity is characterized as a fundamental postulate of quantum mechanics, the resolution of the information problem is vital to complement our current understanding of quantum mechanics. Moreover, its resolution shows that quantum information is never destroyed, even in black holes where the effects of quantum gravity are pronounced. AdS/CFT shows that one can related four-dimensional conformal field theory and five-dimensional Anti-deSitter spacetime. This relation allows us to determine the final state of the black hole after it is radiated by Hawking radiation, which is a cloud of radiation in its pure state. Though the results show that unitarity is conserved in Hawking radiation, further work must be conducted using properties of deSitter spacetime to consolidate them.

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